Sign switching behavior of cross-county interest rate correlations: Theory and Evidence

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Abstract

This paper considers the well established empirical fact that conditional correlations among cross-country interest rates switch signs. Switching implies an alternation of coupling and decoupling of global bond markets over time. This evidence is robust to alternative estimation schemes. Here we use a seminonparametric (SNP) model with a BEKK-GARCH variance function to estimate conditional second moments both to confirm these results and to provide auxiliary models for structural estimation of term structure models. Using an extensive historical analysis, we find that major driving forces behind the sign-switching behavior of conditional correlations between the Eurodollar rate and the Euroyen rate are synchronization and dis-synchronization of business cycles and coordination and discoordination of monetary policies triggered by international policies and financial market crashes. Especially, we find that the two interest rates are more likely to couple when both the U.S. and Japan slip into a recession while the likelihood of decoupling is highest when both economies are in expansion. We also explore whether proposed International Affine Term Structure Models ($\mathbb{IATSM}$s) and International Quadratic Term Structure Models ($\mathbb{IQTSM}$s) are able to reproduce the sign-switching behavior of conditional correlations among cross-country interest rates. We find that a small subset of the $\mathbb{IATSM}$s can generate sign-switching behavior but only by forgoing their ability to describe other features such as the positivity of nominal interest rates, heteroskedasticity in volatility, and correlations among underlying state variables. In contrast, the $\mathbb{IQTSM}$s are able to generate it without limiting their ability to describe other dynamic features. Using the MCMC-Efficient Method of Moments (EMM), we test the empirical performance of the models in reproducing the sign-switching behavior of conditional correlations. The result suggests that the $\mathbb{IATSM}$s conclusively fail to capture it while the $\mathbb{IQTSM}$s are relatively successful but fail to reproduce ephemeral ones.

1 Introduction

At the inception of the subprime mortgage turmoil, "decoupling" gained much credit among practitioners on the grounds that non-U.S. markets spearheaded by the BRICs would be able to stave off financial plague originating in the U.S. and stay on their own growth path. However, this scenario lost credibility as the crisis unfolded, reconfirming the old mantra "if the U.S. sneezes, the rest of the world will catch a cold." Contagion of financial crises across borders, which make seemingly less related markets susceptible to spillover effects, re-emerged as a focal point in gauging the depth, breadth and duration of the crisis. In addition, to save the global economy from the financial crisis and subsequent economic crisis, a majority of governments and central banks orchestrated unprecedented coordination in injecting short-term liquidity, escalating fiscal spending, cutting interest rates, etc. That said, "coupling" may persist during recovery or normalization process let alone the evolution of the crisis itself.

"Coupling and decoupling" of global capital markets is not new. In many respects, the ongoing financial crisis is a 'deja vu' of the Asian currency crisis in 1997 and the subsequent LTCM meltdown in 1998, albeit much more colossal in its size and impact. Since then, there has been a surge in literature exploring driving forces behind synchronization of global asset returns, especially centering upon financial contagion in international *equity* markets.[1] In extant literature, "coupling" and "decoupling" have been addressed primarily in an equity market space, especially by investigating when and how global equity markets' co-movement strengthens and weakens.[2] Those studies suggest strong evidence on time-varying correlations but mixed evidence on the hypothesis that global stock returns tend to be more strongly correlated during crisis periods. More importantly they fail to identify driving forces behind changes in correlations over time.[3]

In contrast, it is only fairly recently that the time-varying correlations, especially their sign-switching behavior, have been explored in the international fixed income security markets.[4] Cappiello, Engle, and Sheppard (2006), *inter alia*, is the first full-scale study on this issue and shows that the correlation between Japanese bond returns and German bond returns plummet to the negative territory at the introduction of the euro, using their asymmetric generalized dynamic conditional correlation (AG-DCC) GARCH model. To confirm their findings, we estimate conditional correlations in interest rates among the U.S. and four countries: Germany, France, Brazil, and Poland.[5] We employ a bivariate seminonparametric (SNP) model with a BEKK-GARCH leading term in the variance function to explore the robustness of the sign-switching behavior.[6] The estimation results are illustrated in Figure 1. Across all the pairs

[1]See King and Wadhwani (1990), Calvo (1999), Kyle and Xiong (2001), Kodres and Pritsker (2002), Yuan (2005), and Brusco and Castiglionesi (2007) among many others and Claessens and Forbes (2001) for a comprehensive review on this topic.

[2]Longin (1996), Straetmans (1998), Baig and Goldfajn (1999), Kamiknsky and Schmukler (1999), Starica (1999), Danielsson and Vries (2000), Forbes and Rigobon (2001), Longin and Solnik (2001), Ang and Chen (2002), and Bae, Karolyi, and Stulz (2003).

[3]See, Bae, Karolyi, and Stulz (2003).

[4]Singleton (1994) addresses the importance of "correlation shocks" in interest rates. However, he does not focus on the sign-switching pattern itself, rather time-varying correlations in non-negative domain.

[5]We use one-year Eurocurrency rate for country pairs of the U.S. and the two developed countries (Germany and France) and two-year government bond rates for country pairs of the U.S. and the two emerging markets (Brazil and Poland) since Eurocurrency rates (Libors) are not available in the emerging markets. The frequency of observation is bi-weekly. Please note that the French and German bond yields became indistinguishable after the Euro was introduced in 2002.

[6]Though not shown here, we also apply an exponentially weighted moving average correlation estimation as an alternative and the results are qualitatively almost identical.

chosen, the correlations' preferred habitat is a positive territory and such a tendency is much stronger between the U.S. and developed countries. However, the correlations occasionally enter into a negative territory, which is followed by either an immediate return to the usual positive habitat or a temporary sojourn in the negative territory. Specifically, the U.S. and German interest rates, which are, on average, more coupled, show negative conditional correlations in 1987 and 1994, albeit ephemerally. In contrast, the conditional correlation between the U.S. and Polish interest rates used to sojourn in a negative domain from late 2007 to early 2008. Thus Figure 1 confirms the empirical findings of Cappiello, Engle, and Sheppard (2006) that the sign-switching behavior is a pronounced feature of cross-country conditional correlations.

On the theoretical side, there has been a surge in international term structure models for the past two decades, since the seminal work of Nielson and Saá-Requejo (1993). Given the fact that these models are an extension of single country term structure models in an international framework, those models have followed progress made in those models. There are two criteria by which the existing international term structure models can be classified. The first criterion is the relationship between underlying state variables and the yields on bonds. Setting aside some exceptions, two representative families have emerged as an outcome: International Affine Term Structure Models (hereafter referred to as $\mathbb{IATSM}$s) and International Quadratic Term Structure Models (hereafter referred to as $\mathbb{IQTSM}$s). The $\mathbb{IATSM}$s inherit the basic framework from the Affine Term structure Models ($\mathbb{ATSM}$ from here on) of Duffie and Kan (1996) and Dai and Singleton (2000), which specify the yield or log bond price as an affine function of underlying state variables. A sequence of $\mathbb{IATSM}$s including Nielson and Saá-Requejo (1993), Saá-Requejo (1993), Bakshi and Chen (1997), Backus, Foresi, and Telmer (2001), Brandt and Stanta-Clara (2002), Ahn and Gao (2003), Ahn (2004), Moburger and Schneider (2005), Benati (2006), Brennan and Xia (2006), and Egorov, Li, and Ng (2009) has been developed. In contrast, the $\mathbb{IQTSM}$s are an international extension of the Quadratic Term Structure Models ($\mathbb{QTSM}$s hereafter) of Ahn, Dittmar, and Gallant (2002), wherein the yield is specified as a quadratic function of state variables. Inci and Lu (2004), Inci (2007), and Leippold and Wu (2007) belong to this family.

A consideration is the inclusion of local factors. Between *a model with common factors only* and *a model with both common and local factors*, it is not clear which one nests which. One might think that a local factor can be regarded as the equivalent of a common factor after restrictions on relevant parameters and that, therefore, the existence of local factors is a testable restriction on a more general model which is composed of common factors only. Unfortunately, this argument is misleading. Ahn (2004) shows that the drift of the stochastic discount factor denominated in a particular country's currency should be driven by its local factor(s) as well as common factors to justify a spanning enhancement from investment in global fixed-income securities. Put differently, one could not expect any enhancement in the investment spanning set by globalizing a fixed-income portfolio in the absence of the local factor(s) specific to the foreign interest rates.[7] Then, would not it be the case that the common and local factor models (which designate the enhancement in the spanning set) should be counted as a more general model than the common factor models (which assume no spanning enhancement in global bond portfolios)? In addition, when the underlying factors themselves are *latent* rather than observable, testing the existence of the local factors by investigating the parametric restrictions could be biased toward rejection since the goodness-of-fit test statistic tends to improve with a more parsimonious parametric

[7]In the absence of the local factors, one can replicate the *currency-hedged* payoff of any foreign bond or bond portfolio by using domestic bonds alone. In addition, if the foreign exchange rate is determined solely by the two countries' term structure factors, even the *naked* payoff of any foreign bond or bond portfolio can be replicated by domestic bonds as well. See Ahn (2004) for the details.

description.[8] Therefore, to a certain extent, introducing domestic and/or foreign local factors is an *a priori* question rather than an empirical question. Combining the two criteria, we can classify the extant models into a two-by-two matrix form: $\mathbb{IATSM}$s with and without local factors and $\mathbb{IQTSM}$s with and without local factors. Most of existing international term structure models belong to one of cells in the matrix.

The primary purpose of our paper is to investigate whether the $\mathbb{IATSM}$s and $\mathbb{IQTSM}$s are theoretically able to generate the sign-switching behavior of cross-country correlations among interest rates. We identify the requisite parametric restrictions and examine the extent to which those restrictions limit the ability of the models to exhibit other important features of international interest rates. To do so, we develop comprehensive $\mathbb{IATSM}$s and $\mathbb{IQTSM}$s that are maximally flexible and thus encompass all features of the models mentioned above. We regard the models with local factors as *a priori* more general given the purpose of our analysis, which is to investigate the behavior of cross-country correlations among interest rates. We identify restrictions on structural parameters that are required to *theoretically* generate the sign-switching dynamics in cross-country interest rate correlations and analyze how restrictive they are. In addition, we also impose restrictions on a factor structure, especially in the $\mathbb{IATSM}$s by eliminating the local factors, to evaluate spanning consequences. These restrictions on the parameters as well as the factor structure are equivalent to sorting out a sub-family of the $\mathbb{IATSM}$s and $\mathbb{IQTSM}$s which can theoretically replicate the sign-switching behavior.

The second purpose of our paper is to estimate those three-factor and four-factor $\mathbb{IATSM}$s and $\mathbb{IQTSM}$s which are theoretically capable of generating sign-switching behavior, using the U.S. and Japan term structure and exchange rate data from 1980 to 2002. We adopt the efficient method of moments (EMM) of Gallant and Tauchen (1996) to estimate those sub-family international term structure models. The EMM is a suitable estimation scheme for our purpose since it enables us to estimate the parameters of the latent stochastic processes within our models and to avoid the discretization bias documented by Aït-Sahalia (1996). Unlike previous empirical studies such as Dai and Singleton (2000) and Ahn, Dittmar, and Gallant (2002), we use the Markov Chain Monte Carlo EMM (MCMC-EMM) proposed by Gallant and Tauchen (2010a, 2010b). The MCMC-EMM has two major innovations over the previous EMM. First, it implements EMM by adopting the MCMC estimator proposed by Chernozhukov and Hong (2003), which is substantially superior to conventional derivative based hill climbing optimizers. Second, the seminonparametric (SNP) auxiliary model uses a BEKK-GARCH specification as the leading term of the conditional variance rather than an R-GARCH leading term. This innovation is particularly important for our analysis since it directly accommodates the estimation of the conditional cross-country correlations among interest rates. An R-GARCH specification has the drawback of losing information on the sign of residuals because the realized residuals enters through the absolute value. The BEKK-GARCH specification does not lose this information. The statistical advantages of EMM as implemented here are that models with latent variables are accommodated and inference is as efficient as maximum likelihood. In addition, test statistics associated with the method allow one to compare non-nested models, to test for lack of fit, and to determine the reason that a model fails to pass the lack of fit test.

The third purpose of our paper is to diagnose the estimated dynamics of conditional correlations

[8] Since the parameters associated with the local factors affect only the relevant country's term structure of interest rates, they are less efficient in *jointly* explaining the two countries' term structures. As a result, when the underlying factors are latent, the goodness-of-fit test will favor uncovering the common factors first rather than the local factors. The underlying intuition is akin to the bias in the estimated eigenvalue (and the resulting factor loadings) of the first factor in a principal component analysis, which is documented in Brown (1989).

from an economic perspective. The estimated dynamics of conditional correlations depends on what conditional moments models are adopted. Recently a number of alternative correlation modeling approaches have been proposed.[9] Despite such progress, it is still an open-ended question whether the estimated time series of conditional correlations reflect a genuine and economically meaningful evolution of correlations that market participants possess. Especially when the correlations sharply fall or rise to the extent that their signs change, it is not clear whether the market participants infer such an *ex ante* break in the second moment or simply whether such a break is an outcome of misspecification of the model adopted or even noise. Herein we evaluate the economic quality of estimated conditional correlations by investigating whether historical events in the U.S. and Japan validate extreme values in conditional correlations and drastic sign-switching thereof. More specifically, we undertake an extensive historical analysis about driving forces behind such extreme levels of or changes in cross-country correlations. We consider business cycle synchronization and coupling in monetary policies (rate cuts and hikes) and market participants' expectation about them as candidates.

The final purpose of our paper is to investigate how well the empirically observed correlations can be tracked by the $\mathbb{IATSM}$s and $\mathbb{IQTSM}$s that are admissible theoretically. In addition, we focus on the ability of these models to capture intermediate swings and ephemeral shocks in correlations.

The remainder of this paper is as follows. In Section 2, we introduce a theoretical framework for international term structure models. Section 3 provides a general characterization and the canonical form of $\mathbb{IATSM}$s and identify restrictions on parameters and a factor structure which are required to theoretically generate the sign-switching behavior of correlations. We undertake a similar analysis for $\mathbb{IQTSM}$s in Section 4. Section 5 provides a discussion of the data and EMM methodology that we use for examining the goodness-of-fit of the $\mathbb{IATSM}$s and the $\mathbb{IQTSM}$s. The empirical results of the EMM estimation and further measurement of the models' fit are also provided in Section 5. Section 6 conducts a historical analysis on driving forces behind extreme values in correlations. The relative performance of the $\mathbb{IATSM}$s and the $\mathbb{IQTSM}$s in reproducing the time series dynamics of correlations is provided in Section 7. We make concluding remarks in Section 8.

2 Theoretical Framework

2.1 The Global Stochastic Discount Factors and Factor Structure

We assume that the world economy is composed of two countries, and is represented by the augmented filtered probability space $(\Omega, F, \mathcal{F}, P)$, where $\mathcal{F} = \{\mathcal{F}_t\}_{0 \leq t \leq \mathcal{T}}$. The uncertainties in the world economy are assumed to be generated by N-independent Brownian motions which

[9] Early approaches such as Bergstrom and Henriksson (1981), Eun and Resnick (1984), and Kaplanis (1988) propose common factor models with Bayesian adjustment. Von Fustenbergt and Jeon (1989) and Erb, Harvey, and Viskanta (1994) adopt a regression approach in describing the time-varying correlations in relation to business cycles. The GARCH-based approach, which is first suggested by Bollerslev (1990), is arguably the most popular. A number of different multivariate GARCH or MGARCH models have been developed. They include the DCC model of Engle (2002), the VEC model of Bollerslev, Engle, and Wooldridge (1988), the R-GARCH model of Gallant and Tauchen (2002), the BEKK model of Engle and Kroner (1995), and the F-GARCH model of Diebold and Nerlove (1989) and Engle, Ng, and Rothschild (1990) to name a few. Please see Palandri (2009) for an extensive survey on the MGARCH models.

have the following form:

$$\begin{aligned} W(t)' &= \left(W_1(t), \cdots, W_{N_c}, W_{N_c+1}(t), \cdots, W_{N_c+N_d}, W_{N_c+N_d+1}(t), \cdots, W_{N_c+N_d+N_f}(t)\right) \\ &\triangleq \left(W^c(t)', W^d(t)', W^f(t)'\right), \end{aligned} \tag{1}$$

where N_c of them are common to both countries, N_d of them are domestic country specific shocks, and N_f of them are foreign country specific shocks, with $N = N_c + N_d + N_f$.

We assume the existence of a positive state-price density, $M_k^k(t)$, in each country, which defines the canonical valuation equation:

$$x_k^k(t) = E_t^P\left[\frac{M_k^k(T)}{M_k^k(t)} x_k^k(T)\right], \tag{2}$$

where $x_k^k(t) : [0, \infty) \times \Omega \to \Re^+$ is the price of a country k (superscript) asset denominated in currency k (superscript). $k = d$ or f stands for *domestic* and *foreign*, respectively. We refer to $M_k^k(t,T) \triangleq \frac{M_k^k(T)}{M_k^k(t)}$ as the stochastic discount factor of country k, which is used to value country k assets denominated in country k's currency. To assert the existence of the unique global SDF, suppose that the global market is complete and permits no arbitrage opportunities. As shown by Ahn (2004), if there is spanning enhancement in globalized portfolios, then the domestic SDF (foreign SDF), $M_d^d(t,T)$ $\left(M_f^f(t,T)\right)$, is extended into the unique global SDF such that

$$\frac{dM_d(t)}{M_d(t)} = \frac{dM_d^d(t)}{M_d^d(t)} - \Lambda_d^f(t)' dW^f(t), \tag{3}$$

$$\frac{dM_f(t)}{M_f(t)} = \frac{dM_f^f(t)}{M_f^f(t)} - \Lambda_f^d(t)' dW^d(t), \tag{4}$$

where $M_d(t,T) \triangleq \frac{M_d(T)}{M_d(t)}$ $\left(M_f(t,T) \triangleq \frac{M_f(T)}{M_f(t)}\right)$ is the unique global SDF defined on domestic (foreign) currency, and $\Lambda_d^f(t) : [0,\infty) \times \Omega \to \Re^{N_f}$ $\left(\Lambda_f^d(t) : [0,\infty) \times \Omega \to \Re^{N_d}\right)$ represents the sources of foreign (domestic) local shocks. Equations (3) and (4) state that when one expects a diversification effect from globalizing one's portfolios, the domestic (foreign) SDF is extended to a globalized one by adding incremental foreign (domestic) local diffusion shocks where the incremental diffusion should not change the drift of the domestic (foreign) SDF.

From (3) and (4), the stochastic differential equations (SDEs) of the global SDFs can be represented generically as:

$$\frac{dM_d(t)}{M_d(t)} = -r_d(t)dt - \Lambda_d(t)' dW(t), \tag{5}$$

$$\frac{dM_f(t)}{M_f(t)} = -r_f(t)dt - \Lambda_f(t)' dW(t), \tag{6}$$

where $\Lambda_d(t)' = \left(\Lambda_d^c(t)', \Lambda_d^d(t)', \Lambda_d^f(t)'\right)$ and $\Lambda_f(t)' = \left(\Lambda_f^c(t)', \Lambda_f^d(t)', \Lambda_f^f(t)'\right)$ are $N \times 1$ vectors of the market prices of common, domestic local, and foreign local factor risks defined on each currency, respectively.

To present the valuation of a discount bond of each country, we make the following assumption about the structure of state variables.

Assumption 1: *Let* $Y(t)' = \left(Y^c(t)', Y^d(t)', Y^f(t)'\right)$ *denotes an* N*-dimensional vector of state variables, where* $Y^c(t)$ *is an* $N_c \times 1$ *vector of common factors,* $Y^d(t)$ *is an* $N_d \times 1$ *vector of domestic local factors, and* $Y^f(t)$ *is an* $N_f \times 1$ *vector of foreign local factors. We assume that* $Y^c(t)$ *can be correlated with* $Y^d(t)$ *and* $Y^f(t)$*. This correlation structure is achieved by allowing* $Y^c(t)$ *to affect the processes of* $Y^d(t)$ *and* $Y^f(t)$*. However, neither local factor can affect the process of* $Y^c(t)$*. Further, we assume that* $Y^d(t)$ *and* $Y^f(t)$ *are orthogonal.*

We can write the time t price of a country k bond that pays one unit of country k currency at maturity date, $T = t + \tau$,

$$P_k(t) = E_t^P \left[\frac{M_k^k(T)}{M_k^k(t)} \right] = E_t^P \left[\frac{M_k(T)}{M_k(t)} \right]. \tag{7}$$

Equation (7) states that the prices of discount bonds are the first moment of the SDF of country k or the global SDF expressed in country k's currency. The extension of the SDF does not make a difference in the valuation of bonds because the second moment of the SDF indirectly affects bond prices via determining term premia. Assumption 1 and the structure of Brownian motions in equation (1) ensure that the common factors can affect both the domestic and foreign bond prices. However, the domestic (foreign) local factors cannot influence the foreign (domestic) bond prices. Thus the two local factors capture the country-specific movements of domestic and foreign bond prices, which cannot be accommodated by the common factors.

From equations (5), (6) and (7), it is clear that specifying a two-country term structure model is equivalent to specifying the stochastic evolution of the global SDFs. In this paper, we directly explore the time series processes of the global SDFs.[10]

2.2 The Dynamics of the Exchange Rate

Since we are simultaneously valuing the bonds of the two countries, the exchange rate is endogenously determined by the no-arbitrage condition. As shown by Backus, Foresi, and Telmer (2001) and Ahn (2004), the depreciation rate of the exchange rate, which does not allow any arbitrate opportunity, is represented as:

$$\frac{S(T)}{S(t)} = \frac{M_f(t,T)}{M_d(t,T)}, \tag{8}$$

where $S(t)$ is the exchange rate defined as the number of units of domestic currency per one unit of foreign currency, and $M_d(t,T)$ and $M_f(t,T)$ are the global SDFs defined on the domestic and foreign currencies, respectively. Applying Ito's lemma to equation (8) leads to the following

[10] This pricing kernel approach is popular in the existing term structure literature. See Constantinides (1992), Ahn and Gao (1999), Dai and Singleton (2000, 2003), Ahn, Dittmar, and Gallant (2002), Bansal and Zhou (2002), and Ahn, Dittmar, Gallant, and Gao (2003) for single-country term structure models. See Backus, Foresi, and Telmer (2001), Brandt and Santa-Clara (2002), Ahn (2004), Inci and Lu (2004), Mosburger and Schneider (2005), Leippold and Wu (2007), and Egorov, Li, and Ng (2009) for two-country term structure models.

SDE of the exchange rate:

$$d\ln S(t) = \left[(r_d(t) - r_f(t)) + \frac{1}{2}\left(\Lambda_d(t)'\Lambda_d(t) - \Lambda_f(t)'\Lambda_f(t)\right)\right]dt + \left(\Lambda_d(t)' - \Lambda_f(t)'\right)dW(t). \quad (9)$$

Equation (9) offers some interesting characteristic of the dynamics of the exchange rate. First, it is clear that the expected depreciation rate is composed of two parts: an interest rate differential and an exchange rate risk premium. If investors of both countries are risk-neutral, the exchange rate is equivalent to the interest rate differential, as stated in the uncovered interest rate parity. However, in a risk-averse world, the foreign exchange rate risk premium, which is a function of the differential of the market prices of factor risks, has a critical role. The exchange rate is determined so as to equalize not only the interest rate differential but also the differences in the market prices of factor risks. Thus the exchange rate risk premium measures the departure from uncovered interest rate parity. Second, the volatility of the log exchange rate is equivalent to the differential of the market prices of factor risks. As such, the foreign exchange rate becomes more volatile as the risk premia differential increases and vice versa. Different assumptions on $\Lambda_d(t)$ and $\Lambda_f(t)$ lead to the different specifications on the exchange rate dynamics.

3 A Characterization of the International Affine Term Structure Models

In this section, we investigate the ability of the $\mathbb{IATSM}$s in a two-country world to generate the sign-switching pattern in cross-country correlations of interest rates. Specifically, we address the following questions:

- Are the $\mathbb{IATSM}$s equipped with an ability of generating *positive* and *negative* (i.e., sign-switching) conditional correlations among the two countries' interest rates?
- If they are, what are the restrictions on parameters and a factor structure required for generating them and how restrictive are they?

To answer the above questions, we first develop comprehensive $\mathbb{IATSM}$s, which are maximally flexible and thus encompasses the features of existing $\mathbb{IATSM}$s. Then we investigate exact restrictions on parameters and a structure of factors that are required to have cross-country conditional correlations with a feasible range from positive to negative. In addition, we examine three-factor and four-factor $\mathbb{IATSM}$s as special cases for empirical estimation.

3.1 A canonical form of $\mathbb{IATSM}$s

In this subsection, we establish a canonical representation of $\mathbb{IATSM}$s which are maximally flexible. Our canonical model for $\mathbb{IATSM}$s can be viewed as an extension of the single-country $\mathbb{ATSM}$s of Dai and Singleton (2000).

To begin with, we summarize the assumptions of an N-factor $\mathbb{IATSM}$. First, the instantaneous interest rate of country k is represented as an affine function of the state variables:

$$r_k(t) = \delta_0^k + \delta_y^{k\prime} Y(t), \quad (10)$$

where δ_0^k is a constant, and δ_y^k is an $N \times 1$ vector of sensitivity of country k's interest rate to state variables, $Y(t)$. Second, the SDEs of the state variables are characterized as an affine diffusion:

$$dY(t) = K(\Theta - Y(t))\,dt + \Sigma\sqrt{S(t)}dW(t), \tag{11}$$

where Θ is an $N \times 1$ vector, K and Σ are $N \times N$ matrices, and $S(t)$ is an N-dimensional diagonal matrix with the i^{th} diagonal term given by

$$[S(t)]_{ii} = \alpha_i + \beta_i' Y(t).$$

For convenience, we stack the β_i vectors into the matrix $\mathbf{B}$, where β_i is the column i of $\mathbf{B}$. Similarly, the scalars α_i are stacked in the $N \times 1$ vector α. Third, we assume that the diffusion term of the global SDF defined on currency k is give by

$$\Lambda_k(t) = \sqrt{S(t)}\lambda^k, \tag{12}$$

where λ^k is an $N \times 1$ vector of constants.

To formalize the family of admissible N-factor $\mathbb{IATSM}$s, we rely on two classification schemes. First, we classify N-factor $\mathbb{IATSM}$s by the number of common, domestic local, and foreign local factors (i.e. N_c, N_d, and N_f). Second, $\mathbb{IATSM}$s with the vector of (N_c, N_d, N_f) are classified into non-nested subfamilies of models by m, the number of state variables that determine $S(t)$. More precisely, the stochastic volatility of $Y^c(t)$, $Y^d(t)$, and $Y^f(t)$ are driven by m_c square-root common factors, $Y^{B_c}(t)$, m_d square-root domestic local factors, $Y^{B_d}(t)$, and m_f square-root foreign local factors, $Y^{B_f}(t)$, respectively, with $m = m_c + m_d + m_f$. Let $\mathbb{IA}_{m;m_c,m_d,m_f}(N; N_c, N_d, N_f)$ denote an N-factor $\mathbb{IATSM}$ that is both admissible and empirically identifiable. We define a canonical representation of $\mathbb{IA}_{m;m_c,m_d,m_f}(N; N_c, N_d, N_f)$ by imposing the factor structure of Assumption 1 on the canonical model of the single-country $\mathbb{ATSM}$s of Dai and Singleton (2000).[11]

Definition 1: *Partitioning Y as*

$$Y' = \left({Y^{B_c}_{m_c \times 1}}', {Y^{B_d}_{m_d \times 1}}', {Y^{B_f}_{m_f \times 1}}', {Y^{D_c}_{(N_c - m_c)\times 1}}', {Y^{D_d}_{(N_d - m_d)\times 1}}', {Y^{D_f}_{(N_f - m_f)\times 1}}' \right),$$

and W as

$$W' = \left({W^{B_c}_{m_c \times 1}}', {W^{B_d}_{m_d \times 1}}', {W^{B_f}_{m_f \times 1}}', {W^{D_c}_{(N_c - m_c)\times 1}}', {W^{D_d}_{(N_d - m_d)\times 1}}', {W^{D_f}_{(N_f - m_f)\times 1}}' \right),$$

we define the canonical representation of $\mathbb{IA}_{m;m_c,m_d,m_f}(N; N_c, N_d, N_f)$ by adding the following restrictions on equations (10) and (11):

$$\delta_y^{d\prime} = \left({\delta^d_{y^{B_c}}}', {\delta^d_{y^{B_d}}}', 0', {\delta^d_{y^{D_c}}}', {\delta^d_{y^{D_d}}}', 0' \right), \tag{13}$$

$$\delta_y^{f\prime} = \left({\delta^f_{y^{B_c}}}', 0', {\delta^f_{y^{B_f}}}', {\delta^f_{y^{D_c}}}', 0', {\delta^f_{y^{D_f}}}' \right), \tag{14}$$

$$K = \begin{bmatrix} K^{BB}_{m \times m} & 0_{m \times (N-m)} \\ K^{DB}_{(N-m)\times m} & K^{DD}_{(N-m)\times(N-m)} \end{bmatrix}, \tag{15}$$

[11]Put differently, we extend the terminology of $A_m(N)$ of Dai and Singleton (2000) into a two-country setup. In a single-country, m square-root factors drive the stochastic volatility of $Y(t)$.

where

$$K^{BB} = \begin{bmatrix} K^{B_cB_c}_{m_c\times m_c} & 0_{m_c\times m_d} & 0_{m_c\times m_f} \\ K^{B_dB_c}_{m_d\times m_c} & K^{B_dB_d}_{m_d\times m_d} & 0_{m_d\times m_f} \\ K^{B_fB_c}_{m_f\times m_c} & 0_{m_f\times m_d} & K^{B_fB_f}_{m_f\times m_f} \end{bmatrix}, \tag{16}$$

$$K^{DB} = \begin{bmatrix} K^{D_cB_c}_{(N_c-m_c)\times m_c} & 0_{(N_c-m_c)\times m_d} & 0_{(N_c-m_c)\times m_f} \\ K^{D_dB_c}_{(N_d-m_d)\times m_c} & K^{D_dB_d}_{(N_d-m_d)\times m_d} & 0_{(N_d-m_d)\times m_f} \\ K^{D_fB_c}_{(N_f-m_f)\times m_c} & 0_{(N_f-m_f)\times m_d} & K^{D_fB_f}_{(N_f-m_f)\times m_f} \end{bmatrix}, \tag{17}$$

$$K^{DD} = \begin{bmatrix} K^{D_cD_c}_{(N_c-m_c)\times(N_c-m_c)} & 0_{(N_c-m_c)\times(N_d-m_d)} & 0_{(N_c-m_c)\times(N_f-m_f)} \\ K^{D_dD_c}_{(N_d-m_d)\times(N_c-m_c)} & K^{D_dD_d}_{(N_d-m_d)\times(N_d-m_d)} & 0_{(N_d-m_d)\times(N_f-m_f)} \\ K^{D_fD_c}_{(N_f-m_f)\times(N_c-m_c)} & 0_{(N_f-m_f)\times(N_d-m_d)} & K^{D_fD_f}_{(N_f-m_f)\times(N_f-m_f)} \end{bmatrix}, \tag{18}$$

for $m > 0$*, and* $K^{D_cD_c}$*,* $K^{D_dD_d}$*, and* $K^{D_fD_f}$ *are lower triangular matrices for* $m = 0$*,*

$$\Theta' = \left(\Theta^{B_c}_{m_c\times 1}{}', \Theta^{B_d}_{m_d\times 1}{}', \Theta^{B_f}_{m_f\times 1}{}', 0_{(N_c-m_c)\times 1}{}', 0_{(N_d-m_d)\times 1}{}', 0_{(N_f-m_f)\times 1}{}'\right), \tag{19}$$

$$\Sigma = \begin{bmatrix} I_{m\times m} & 0_{m\times(N-m)} \\ 0_{(N-m)\times m} & I_{(N-m)\times(N-m)} \end{bmatrix}, \tag{20}$$

$$\alpha' = \left(0_{m\times 1}{}', 1_{(N-m)\times 1}{}'\right), \tag{21}$$

$$\mathbf{B} = \begin{bmatrix} I_{m\times m} & \mathbf{B}^{BD}_{m\times(N-m)} \\ 0_{(N-m)\times m} & 0_{(N-m)\times(N-m)} \end{bmatrix}, \tag{22}$$

where

$$\mathbf{B}^{BD} = \begin{bmatrix} \mathbf{BB}^{B_cD_c}_{m_c\times(N_c-m_c)} & \mathbf{B}^{B_cD_d}_{m_c\times(N_d-m_d)} & \mathbf{B}^{B_cD_f}_{m_c\times(N_f-m_f)} \\ 0_{m_d\times(N_c-m_c)} & \mathbf{B}^{B_dD_d}_{m_d\times(N_d-m_d)} & 0_{m_d\times(N_f-m_f)} \\ 0_{m_f\times(N_c-m_c)} & 0_{m_f\times(N_d-m_d)} & \mathbf{B}^{B_fD_f}_{m_f\times(N_f-m_f)} \end{bmatrix}; \tag{23}$$

with the parametric restrictions described in Appendix B.

In our canonical model, the short rate process of each country is driven by the common and country-specific local factors. From equations (13) and (14), the domestic interest rate, $r_d(t)$, is determined by the common factors, $Y^c(t)$, and the domestic local factors, $Y^d(t)$. Similarly, only the common and foreign local factors, $Y^c(t)$ and $Y^f(t)$, determine the foreign interest rate, $r_f(t)$. Assumption 1 is required to prevent the domestic (foreign) local factors from indirectly affecting the foreign (domestic) interest rate. Restricting $K^{B_cB_d}$ ($K^{B_cB_f}$), $K^{D_cB_d}$ ($K^{D_cB_f}$), and $K^{D_cD_d}$ ($K^{D_cD_f}$) to zero assures that $Y^d(t)$ ($Y^f(t)$) cannot affect the drift of $Y^c(t)$. The diffusion term of $Y^c(t)$ is independent of $Y^d(t)$ ($Y^f(t)$) by restricting $B^{B_dD_c}$ ($B^{B_fD_c}$) to zero. The independence of $Y^d(t)$ and $Y^f(t)$ is ensured by zero restrictions on $K^{B_fB_d}$, $K^{D_fB_d}$, $K^{D_fD_d}$, $B^{B_dD_f}$, $K^{B_dB_f}$, $K^{D_dB_f}$, $K^{D_dD_f}$, and $B^{B_fD_d}$.[12]

[12]The identifiability conditions for $\mathbb{IATSM}$s are more flexible than those of the single-country $\mathbb{ATSM}$s. This feature arises from the co-existence of domestic and foreign bonds. Identifying the level of $Y(t)$ by normalizing $\alpha_i = 0$, $0 \le i \le m$, and $\Theta_i = 0$, $m+1 \le i \le N$, enables us to identify δ_0^d, δ_0^f, and $\Theta_i = 0$, $1 \le i \le m$. Some model parameters that govern the dynamics of the common factors are identified by exploiting either the domestic or the foreign bond prices. First, by fixing the scale of $Y^{B_c}(t)$ ($Y^{D_c}(t)$) through $\mathbf{B}_{ii} = 1$, $1 \le i \le m$, ($\alpha_i = 1$,

3.2 An ability of $\mathbb{IATSM}$s to generate the sign-switching correlations

Here we investigate the characteristics of the $\mathbb{IATSM}$s to identify the subfamilies of models that are capable of generating the sign switching behavior. Our canonical model enables us to investigate the correlation generating mechanism of $\mathbb{IATSM}$s. From Ito's lemma, the instantaneous covariance of the two-country interest rates is represented as:

$$\text{Cov}_{df}(t) = \delta^{d\,\prime}_{y^{B_c}} S^{B_c}(t)\delta^f_{y^{B_c}} + \delta^{d\,\prime}_{y^{D_c}} S^{D_c}(t)\delta^f_{y^{D_c}}, \tag{24}$$

where $S^{B_c}(t)$ and $S^{D_c}(t)$ are m_c- and $(N_c - m_c)$-dimensional diagonal matrices with the elements on the main diagonal given by $[S^{B_c}(t)]_{ii} = [Y^{B_c}(t)]_i$, and $[S^{D_c}(t)]_{ii} = 1 + [B^{B_cD_c\prime}Y^{B_c}(t)]_i$, respectively. It is clear that the covariance of the interest rates is an affine function of the square-root common factors, $Y^{B_c}(t)$, which can take only positive values. Therefore an increase in the number, m_c, of square-root common factors gives more flexibility in modeling the sign-switching correlation of the interest rates, $r_d(t)$ and $r_f(t)$. From equation (24), we can identify the following necessary condition for $\mathbb{IATSM}$s to generate the sign-switching property of the correlation.

Condition 1: (Conditions for sign-switching correlations)
(a) $m_c \geq 1$ *for* $N_c - m_c \geq 1$, *and* $m_c \geq 2$ *for* $N_c - m_c = 0$,
(b) For some $1 \leq i \leq m_c$, $\left[\delta^d_{y^{B_c}} \circ \delta^f_{y^{B_c}}\right]_i < 0$,
(c) For some $1 \leq i \leq N_c - m_c$, $\left[\delta^d_{y^{D_c}} \circ \delta^f_{y^{D_c}}\right]_i < 0$,
where $\circ$ *is a Hadamard product.*

Part (a) of Condition 1, the proof of which is proved in Appendix C, states that we need one or more square-root common factors, $Y^{B_c}(t)$, when there exist Gaussian common factors, $Y^{D_c}(t)$. However, in the case of the subfamilies of the models without a Gaussian common factor, there must be at least *two* square-root common factors to generate the sign-switching correlation. $\mathbb{IATSM}$s satisfying part (a) are theoretically capable of accommodating the sign-switching property of the correlation through parts (b) and (c). These parts of Condition 1 indicate that the instantaneous correlation of the interest rates can switch sign over time through the opposite signs of the sensitivities of the interest rates to the square-root common factors, $Y^{B_c}(t)$, and to the Gaussian common factors, $Y^{D_c}(t)$.

Based on equation (24) and Condition 1, we can establish the following propositions regarding the characteristics of $\mathbb{IATSM}$s.

Proposition 1: *Any subfamily of* $\mathbb{IA}_{m;m_c,m_d,m_f}(N; N_c, N_d, N_f)$ *with* $N_c = m_c$, $N_d = m_d$, *and* $N_f = m_f$ *cannot generate the sign-switching correlation of* $r_d(t)$ *and* $r_f(t)$ *without violating the positivity of* $r_d(t)$ *and* $r_f(t)$.
Proof. See Appendix D.

The only family of $\mathbb{IATSM}$s that ensures positive interest rates are those in the family of $\mathbb{IA}_{m;m_c,m_d,m_f}(m; m_c, m_d, m_f)$. Unfortunately this family of models cannot accommodate the

$m+1 \leq i \leq N$), and $\Sigma_{ii} = 1$, $1 \leq i \leq N$, we can identify $\delta^d_{y^{B_c}}$ $(\delta^d_{y^{D_c}})$ and $\delta^f_{y^{B_c}}$ $(\delta^f_{y^{D_c}})$. Second, the sign of $Y^{D_c}(t)$ is determined once we impose the normalization that $\delta^d_{y^{D_c}} \geq 0$. The remaining identifiability conditions are the same as those of the single-country $\mathbb{ATSM}$s provided by Dai and Singleton (2000).

negative correlation of $r_d(t)$ and $r_f(t)$.[13] As a result, $\mathbb{IATSM}$s cannot simultaneously allow for the sign-switching cross-country correlation and guarantee the positivity of the nominal interest rates. In addition, as noted by Dai and Singleton (2000), admissibility of an $\mathbb{ATSM}$ requires non-negative correlations among the square-root factors. Thus $\mathbb{IA}_{m;m_c,m_d,m_f}(m; m_c, m_d, m_f)$ cannot accommodate negative correlations among the state variables.

Proposition 2: *Any subfamily of* $\mathbb{IA}_{m;m_c,m_d,m_f}(N; N_c, N_d, N_f)$ *with* $m_c = m_d = m_f = 0$ *cannot induce the stochastic correlation of* $r_d(t)$ *and* $r_f(t)$.
Proof. See Appendix E.

This proposition states that the Gaussian factor models, which are the most flexible $\mathbb{IATSM}$s in specifying conditional/unconditional correlations among the state variables, cannot accommodate the sign-switching correlation of $r_d(t)$ and $r_f(t)$. Gaussian factor models are able to induce only homoskedastic correlation of $r_d(t)$ and $r_f(t)$, whose sign can be either positive or negative.

Proposition 3: *Any subfamily of* $\mathbb{IA}_{m;m_c,m_d,m_f}(N; N_c, N_d, N_f)$ *with* $N_c + N_d + N_f = 3$ *cannot simultaneously generate the sign-switching correlation and the country-specific movements of* $r_d(t)$ *and* $r_f(t)$.
Proof. See Appendix F.

This undesirable property of $\mathbb{IATSM}$s immediately follows from part (a) of Condition 1. As a result, we need at least four factors to model both the sign-switching correlation and the country-specific movements of the interest rates.

We turn next to the valuation of bonds. Following Dai and Singleton (2000), and using the results in equations (5) and (6), the diffusion term of the global SDF defined on currency k is assume to be

$$\Lambda_k(t) = \sqrt{S(t)}\lambda^k, \tag{25}$$

where

$$\lambda^{k\prime} = \left(\lambda^k_{y^{B_c}}{}', \lambda^k_{y^{B_d}}{}', \lambda^k_{y^{B_f}}{}', \lambda^k_{y^{D_c}}{}', \lambda^k_{y^{D_d}}{}', \lambda^k_{y^{D_f}}{}'\right). \tag{26}$$

The price of a discount bond is designated as an exponential affine function of the state variables:

$$P_k(t,\tau) = exp\left[A^k(\tau) + B^k(\tau)'Y(t)\right], \tag{27}$$

where

$$B^d(\tau)' = \left(B^d_{y^{B_c}}(\tau)', B^d_{y^{B_d}}(\tau)', 0', B^d_{y^{D_c}}(\tau)', B^d_{y^{D_d}}(\tau)', 0'\right),$$
$$B^f(\tau)' = \left(B^f_{y^{B_c}}(\tau)', 0', B^f_{y^{B_f}}(\tau)', B^f_{y^{D_c}}(\tau)', 0', B^f_{y^{D_f}}(\tau)'\right).$$

Thus the domestic (foreign) bond price, $P_d(t,\tau)$ ($P_f(t,\tau)$), is not affected by the foreign (domestic) local factors, $Y^f(t)$ ($Y^d(t)$). Duffie and Kan (1996) show that $A^k(\tau)$ and $B^k(t,\tau)$ satisfy

[13] As demonstrated by Ahn (2004), when there exist no local factors, the feasible range of the correlation of $r_d(t)$ and $r_f(t)$ is (0,1]. The existence of local factors in $IA_{m;m_c,m_d,m_f}(m; m_c, m_d, m_f)$ results in the lower bound of zero.

the following ordinary differential equations (ODEs):

$$\begin{aligned}\frac{dA^k(\tau)}{d\tau} &= -\widetilde{\Theta}_k'\widetilde{K}_k'B^k(\tau) + \frac{1}{2}\sum_{i=1}^{N}\left[\Sigma' B^k(\tau)\right]_i^2 \alpha_i - \delta_0^k, \\ \frac{dB^k(\tau)}{d\tau} &= -\widetilde{K}_k'B^k(\tau) - \frac{1}{2}\sum_{i=1}^{N}\left[\Sigma' B^k(\tau)\right]_i^2 \beta_i + \delta_y^k,\end{aligned}$$

with the initial conditions $A^k(0) = 0_{1\times 1}$ and $B^k(0) = 0_{N\times 1}$. $\widetilde{K}_k = K + \Sigma\Xi_k$ and $\widetilde{\Theta}_k = \widetilde{K}_k^{-1}[K\Theta - \Sigma\xi_k]$ are the parameters of the SDEs of the state variables under the equivalent martingale measure defined on currency k, where Ξ_k is an $N\times N$ matrix with the i^{th} row given by $\beta_i'[\lambda^k]_i$, and ξ_k is an $N\times 1$ vector with the i^{th} element given by $\alpha[\lambda^k]_i$.

The yield-to-maturity, $yt_k(t,\tau)$, is defined as $-\ln P_k(t,\tau)/\tau$:

$$yt_k(t,\tau) = \frac{1}{\tau}\left[-\ln A^k(\tau) - B^k(\tau)'Y(t)\right].$$

It can be shown that the conditional correlation of $yt_d(t,\tau)$ and $yt_f(t,\tau)$ is an affine function of the square-root common factors, $Y^{B_c}(t)$. As a result, $\mathbb{IATSM}$s can accommodate the sign-switching correlation of $yt_d(t,\tau)$ and $yt_f(t,\tau)$ through $\left[B^d_{y^{B_c}}(\tau)\circ B^f_{y^{B_c}}(\tau)\right]_i < 0$ and $\left[B^d_{y^{D_c}}(\tau)\circ B^f_{y^{D_c}}(\tau)\right]_j < 0$ for some $1\le i\le m_c$ and $1\le j\le N_c - m_c$.

3.3 Reconciliation of sign-switching correlation and negative interest rates

Under parts (b) and (c) of Condition 1, there is a positive probability that the two interest rates, $r_d(t)$ and $r_t(t)$, may stay in negative domain for a prolonged period. In this subsection, we explore the condition under which the sign-switching correlation of the interest rates is admissible while not allowing for permanently negative interest rates. This condition is identified by restricting the signs of $\delta^d_{y^{B_c}}$, $\delta^f_{y^{B_c}}$, and $\delta^f_{y^{D_c}}$ to be non-negative.[14]

In equations (15) and (20), normalizing K^{DB} to zero and K^{DD} to a diagonal matrix, we can free up Σ^{DB} and Σ^{DD} such that

$$\Sigma^{DB} = \begin{bmatrix} \Sigma^{D_cB_c}_{(N_c-m_c)\times m_c} & 0_{(N_c-m_c)\times m_d} & 0_{(N_c-m_c)\times m_f} \\ \Sigma^{D_dB_c}_{(N_d-m_d)\times m_c} & \Sigma^{D_dB_d}_{(N_d-m_d)\times m_d} & 0_{(N_d-m_d)\times m_f} \\ \Sigma^{D_fB_c}_{(N_f-m_f)\times m_c} & 0_{(N_f-m_f)\times m_d} & \Sigma^{D_fB_f}_{(N_f-m_f)\times m_f} \end{bmatrix},$$

$$\Sigma^{DD} = \begin{bmatrix} \Sigma^{D_cD_c}_{(N_c-m_c)\times(N_c-m_c)} & 0_{(N_c-m_c)\times(N_d-m_d)} & 0_{(N_c-m_c)\times(N_f-m_f)} \\ \Sigma^{D_dD_c}_{(N_d-m_d)\times(N_c-m_c)} & \Sigma^{D_dD_d}_{(N_d-m_d)\times(N_d-m_d)} & 0_{(N_d-m_d)\times(N_f-m_f)} \\ \Sigma^{D_fD_c}_{(N_f-m_f)\times(N_c-m_c)} & 0_{(N_f-m_f)\times(N_d-m_d)} & \Sigma^{D_fD_f}_{(N_f-m_f)\times(N_f-m_f)} \end{bmatrix},$$

[14] Note that the signs of $\delta^d_{y^{D_c}}$, $\delta^d_{y^{D_d}}$, and $\delta^f_{y^{D_f}}$ are normalized to be non-negative in order to identify the signs of the Gaussian factors, $Y^{D_c}(t)$, $Y^{D_d}(t)$, and $Y^{D_f}(t)$, as presented in Appendix A.

where the diagonal elements of $\Sigma^{D_cD_c}$, $\Sigma^{D_dD_d}$, and $\Sigma^{D_fD_f}$ are normalized to 1s. This normalization yields an equivalent form to our model of $\mathbb{IA}_{m;m_c,m_d,m_f}(N;N_c,N_d,N_f)$ and leads to the following representation of the covariance of $r_d(t)$ and $r_d(t)$:

$$\begin{aligned}\mathrm{Cov}_{df}(t) &= \left(\delta^{d}_{y^{B_c}}{}' + \delta^{d}_{y^{D_c}}{}'\Sigma^{D_cB_c} + \delta^{d}_{y^{D_d}}{}'\Sigma^{D_dB_c}\right) S^{B_c}(t) \left(\Sigma^{D_fB_c}{}'\delta^{f}_{y^{D_f}} + \Sigma^{D_cB_c}{}'\delta^{f}_{y^{D_c}} + \delta^{f}_{y^{B_c}}\right) \\ &+ \left(\delta^{d}_{y^{D_c}}{}'\Sigma^{D_cD_c} + \delta^{d}_{y^{D_d}}{}'\Sigma^{D_dD_c}\right) S^{D_c}(t) \left(\Sigma^{D_fD_c}{}'\delta^{f}_{y^{D_f}} + \Sigma^{D_cD_c}{}'\delta^{f}_{y^{D_c}}\right). \qquad (28)\end{aligned}$$

Equation (28) states $\mathbb{IATSM}$s satisfying part(a) of Condition 1 are capable of accommodating the sign-switching correlation of $r_d(t)$ and $r_f(t)$ only through the negative correlations among the state variables, which are captured by $\Sigma^{D_cB_c}$, $\Sigma^{D_dB_c}$, $\Sigma^{D_fB_c}$, $\Sigma^{D_dD_c}$, $\Sigma^{D_fD_c}$, and the off-diagonal terms of $\Sigma^{D_cD_c}$. Therefore, excluding sustained negative interest rates requires a trade-off between flexibility in generating the sign-switching correlation of the interest rates and in accommodating heteroskedasticity of the conditional second moments of the interest rates.[15]

3.4 Three and four factor $\mathbb{IATSM}$s considered

In this section, we specify each of the $\mathbb{IATSM}$s that we estimate and discuss their implication for the sign-switching correlation of the cross-country interest rates. Since our primary interest lies in the capability of $\mathbb{IATSM}$s to capture the sign-switching correlation of the two-country interest rates and bond prices, we investigate models wherein the sign-switching property is admissible. Further, we focus only on three- and four-factor models. Unlike a single-country setup, there are still ongoing debates on the number of factors governing international term structure of interest rates.[16] However, considering our empirical investigation presented later, in which we utilize two yields and exchange rate return data, our choice of N is not restrictive.

As presented in Table 1, there are eight subfamilies of three-factor $\mathbb{IATSM}$s, while maintaining the symmetry of the factor structure of each local market. Among these models, we investigate $\mathbb{IA}_{1;1,0,0}(3;3,0,0)$ and $\mathbb{IA}_{2;2,0,0}(3;3,0,0)$, which are capable of accommodating the sign-switching property of the cross-country interest rate correlation. According to Proposition 3, any three-factor $\mathbb{IATSM}$ including local factors is theoretically incapable of generating the sign-switching correlation. In contrast, both $\mathbb{IA}_{1;1,0,0}(3;3,0,0)$ and $\mathbb{IA}_{2;2,0,0}(3;3,0,0)$ cannot accommodate the country-specific dynamics of the interest rates because they include common factors only. To consider the models that are able to simultaneously induce the sign-switching correlation and the country-specific movements of the interest rates, we consider four-factor models. As shown by Table 1, there are six subfamilies of four-factor $\mathbb{IATSM}$s which include both common and local

[15] Mosburger and Schneider (2005) and Egorov, Li, and Ng (2009) demonstrate that there is a trade-off in specifying the heteroskedastic volatility and the negative correlation of the two interest rates, $r_d(t)$ and $r_f(t)$. However, these papers do not address the sign-switching property of the cross-country interest rate correlations.

[16] In a single-country analysis, especially in U.S., many existing studies stipulate a coherent empirical finding regarding the number of factors: three factors are known to be good enough for describing the movement of yield curve [See, e.g. Litterman and Scheinkman (1991), Dai and Singleton (2000), Ahn, Dittmar, and Gallant (2002), Duffee (2002), and Ahn, Dittmar, Gallant, and Gao (2003)]. To model the joint term structure of interest rates of U.S. and Germany, Ahn (2004) provides a three-factor affine model. Similarly, Mosburger and Schneider (2005) develop three-factor affine models to investigate the term structure of U.S. and U.K. simultaneously. Egorov, Li, and Ng (2009) model the joint term structure of U.S. and E.U. using four-factor affine models. Inci and Lu (2004) investigate their three- and five-factor two-country quadratic models using U.S.-U.K. and U.S.-Germany data. Leippold and Wu (2007) focus on their six-factor quadratic model in modeling the joint term structure of U.S. and Japan.

factors. We investigate the performance of $\mathbb{IA}_{2;2,0,0}(4;2,1,1)$ and $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$, which are most flexible in generating the sign-switching correlation of r_d and r_f.[17] Assumption 1 implies that each four-factor $\mathbb{IATSM}$ in Table 1 collapses to a three-factor single-country $\mathbb{ATSM}$. In the case of $\mathbb{IA}_{2;2,0,0}(4;2,1,1)$, the domestic (foreign) interest rate, $r_d(t)$ $(r_f(t))$, is determined by the two square-root common factors, $Y_1(t)$ and $Y_2(t)$, and by the Gaussian domestic (foreign) local factor, $Y_3(t)$ $(Y_4(t))$. Therefore both the domestic and foreign local markets are described by $A_2(3)$. Similarly, $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$ collapses to a restricted version of $A_2(3)$, wherein the dynamics of $r_d(t)$ $(r_f(t))$ is governed by the two square-root factors, $Y_1(t)$ and $Y_2(t)$ $(Y_3(t))$, and the Gaussian common factor, $Y_4(t)$.

(1) Three factor $\mathbb{IATSM}$s

In both models $\mathbb{IA}_{1;1,0,0}(3;3,0,0)$ and $\mathbb{IA}_{2;2,0,0}(3;3,0,0)$, the two short rates are designated as affine functions of three common factors, $Y_1(t), Y_2(t),$ and $Y_3(t)$, such that

$$r_d(t) = \delta_0^d + \delta_1^d Y_1(t) + \delta_2^d Y_2(t) + \delta_3^d Y_3(t), \tag{29}$$

$$r_f(t) = \delta_0^f + \delta_1^f Y_1(t) + \delta_2^f Y_2(t) + \delta_3^f Y_3(t). \tag{30}$$

$\mathbb{IA}_{1;1,0,0}(3;3,0,0)$

In this model, the underlying factor processes are governed by one square-root common factor, $Y_1(t)$, and two Gaussian common factors, $Y_2(t)$ and $Y_3(t)$, which are assumed to follow the following stochastic processes:

$$\begin{aligned} d\begin{pmatrix} Y_1(t) \\ Y_2(t) \\ Y_3(t) \end{pmatrix} &= \begin{pmatrix} \kappa_{11} & 0 & 0 \\ \kappa_{21} & \kappa_{22} & \kappa_{23} \\ \kappa_{31} & \kappa_{32} & \kappa_{33} \end{pmatrix} \begin{pmatrix} \theta_1 & - & Y_1(t) \\ & - & Y_2(t) \\ & - & Y_3(t) \end{pmatrix} dt \\ &+ \begin{pmatrix} \sqrt{Y_1(t)} & 0 & 0 \\ 0 & \sqrt{1+\beta_{21}Y_1(t)} & 0 \\ 0 & 0 & \sqrt{1+\beta_{31}Y_1(t)} \end{pmatrix} dW(t). \end{aligned} \tag{31}$$

Applying Ito's lemma to equations (29) and (30) results in the following SDEs of the interest rates:

$$\begin{aligned} dr_d(t) &= \left[\sum_{i=1}^{3} \delta_i^d \kappa_{i1} \left(\theta_1 - Y_1(t)\right) - \sum_{i=2}^{3}\sum_{j=2}^{3} \delta_i^d \kappa_{ij} Y_j(t)\right] dt \\ &+ \delta_1^d \sqrt{Y_1(t)} dW_1(t) + \delta_2^d \sqrt{1+\beta_{21}Y_1(t)} dW_2(t) + \delta_3^d \sqrt{1+\beta_{31}Y_1(t)} dW_3(t), \end{aligned} \tag{32}$$

$$\begin{aligned} dr_f(t) &= \left[\sum_{i=1}^{3} \delta_i^f \kappa_{i1} \left(\theta_1 - Y_1(t)\right) - \sum_{i=2}^{3}\sum_{j=2}^{3} \delta_i^f \kappa_{ij} Y_j(t)\right] dt \\ &+ \delta_1^f \sqrt{Y_1(t)} dW_1(t) + \delta_2^f \sqrt{1+\beta_{21}Y_1(t)} dW_2(t) + \delta_3^f \sqrt{1+\beta_{31}Y_1(t)} dW_3(t). \end{aligned} \tag{33}$$

[17] From part (a) of Condition 1, $\mathbb{IA}_{2;0,1,1}(4;2,1,1)$ is incapable of generating the sign-switching correlation of $r_d(t)$ and $r_f(t)$.

Thus the instantaneous correlation of the two-country interest rates is represented as:

$$\mathrm{Corr}_{df}(t) = \frac{\mathrm{Cov}_{df}(t)}{\sqrt{\mathrm{Var}_d(t)}\sqrt{\mathrm{Var}_f(t)}}, \tag{34}$$

where

$$\begin{aligned}
\mathrm{Var}_d(t) &= \delta_1^{d2} Y_1(t) + \delta_2^{d2}\left(1 + \beta_{21} Y_1(t)\right) + \delta_3^{d2}\left(1 + \beta_{31} Y_1(t)\right), \\
\mathrm{Var}_f(t) &= \delta_1^{f2} Y_1(t) + \delta_2^{f2}\left(1 + \beta_{21} Y_1(t)\right) + \delta_3^{f2}\left(1 + \beta_{31} Y_1(t)\right), \\
\mathrm{Cov}_{df}(t) &= \delta_1^d \delta_1^f Y_1(t) + \delta_2^d \delta_2^f \left(1 + \beta_{21} Y_1(t)\right) + \delta_3^d \delta_3^f \left(1 + \beta_{31} Y_1(t)\right).
\end{aligned}$$

Equation (34) states that the correlation is a function of the square-root common factor, $Y_1(t)$, which cannot switch sign over time. Thus a necessary condition to generate the sign-switching correlation is that of one or two terms of $(\delta_1^d\delta_1^f, \delta_2^d\delta_2^f, \delta_3^d\delta_3^f)$ must take negative value.

$\underline{\mathbb{IA}_{2;2,0,0}(3;3,0,0)}$

The subfamily $\mathbb{IA}_{2;2,0,0}(3;3,0,0)$ is based on two square-root common factors, $Y_1(t)$ and $Y_2(t)$, and one Gaussian common factor, $Y_3(t)$, the SDEs of which are represented as:

$$\begin{aligned}
d\begin{pmatrix} Y_1(t) \\ Y_2(t) \\ Y_3(t) \end{pmatrix} &= \begin{pmatrix} \kappa_{11} & \kappa_{12} & 0 \\ \kappa_{21} & \kappa_{22} & 0 \\ \kappa_{31} & \kappa_{32} & \kappa_{33} \end{pmatrix} \begin{pmatrix} \theta_1 & - & Y_1(t) \\ \theta_2 & - & Y_2(t) \\ & - & Y_3(t) \end{pmatrix} dt \\
&+ \begin{pmatrix} \sqrt{Y_1(t)} & 0 & 0 \\ 0 & \sqrt{Y_2(t)} & 0 \\ 0 & 0 & \sqrt{1 + \beta_{31} Y_1(t) + \beta_{32} Y_2(t)} \end{pmatrix} dW(t).
\end{aligned} \tag{35}$$

By Ito's lemma, the SDEs of the interests become

$$\begin{aligned}
dr_d(t) &= \left[\sum_{i=1}^{3} \delta_i^d \kappa_{i1}\left(\theta_1 - Y_1(t)\right) - \sum_{i=1}^{3} \delta_i^d \kappa_{i2}\left(\theta_2 - Y_2(t)\right) - \delta_3^d \kappa_3 3 Y_3(t)\right] dt \\
&+ \delta_1^d \sqrt{Y_1(t)} dW_1(t) + \delta_2^d \sqrt{Y_2(t)} dW_2(t) + \delta_3^d \sqrt{1 + \beta_{31} Y_1(t) + \beta_{32} Y_2(t)} dW_3(t),
\end{aligned} \tag{36}$$

$$\begin{aligned}
dr_f(t) &= \left[\sum_{i=1}^{3} \delta_i^f \kappa_{i1}\left(\theta_1 - Y_1(t)\right) - \sum_{i=1}^{3} \delta_i^f \kappa_{i2}\left(\theta_2 - Y_2(t)\right) - \delta_3^f \kappa_3 3 Y_3(t)\right] dt \\
&+ \delta_1^f \sqrt{Y_1(t)} dW_1(t) + \delta_2^f \sqrt{Y_2(t)} dW_2(t) + \delta_3^f \sqrt{1 + \beta_{31} Y_1(t) + \beta_{32} Y_2(t)} dW_3(t).
\end{aligned} \tag{37}$$

Thus the instantaneous correlation of the interest rates is represented as:

$$\mathrm{Corr}_{df}(t) = \frac{\mathrm{Cov}_{df}(t)}{\sqrt{\mathrm{Var}_d(t)}\sqrt{\mathrm{Var}_f(t)}}, \tag{38}$$

where

$$\begin{aligned}
\mathrm{Var}_d(t) &= \delta_1^{d2} Y_1(t) + \delta_2^{d2} Y_2(t) + \delta_3^{d2}\left(1 + \beta_{31} Y_1(t) + \beta_{32} Y_2(t)\right), \\
\mathrm{Var}_f(t) &= \delta_1^{f2} Y_1(t) + \delta_2^{f2} Y_2(t) + \delta_3^{f2}\left(1 + \beta_{31} Y_1(t) + \beta_{32} Y_2(t)\right), \\
\mathrm{Cov}_{df}(t) &= \delta_1^d \delta_1^f Y_1(t) + \delta_2^d \delta_2^f Y_2(t) + \delta_3^d \delta_3^f \left(1 + \beta_{31} Y_1(t) + \beta_{32} Y_2(t)\right).
\end{aligned}$$

It is clear that stochastic correlation is governed by the two square-root factors, $Y_1(t)$ and $Y_2(t)$. To accommodate the sign-switching property of the correlation of $r_d(t)$ and $r_d(t)$, the signs of one or two terms of $(\delta_1^d\delta_1^f, \delta_2^d\delta_2^f, \delta_3^d\delta_3^f)$ must be negative.

(2) Four factor $\mathbb{IATSM}$s

$\mathbb{IA}_{2;2,0,0}(4;2,1,1)$

This model is characterized by two square-root common factors, $Y_1(t)$ and $Y_2(t)$, one Gaussian domestic local factor, $Y_3(t)$, and one Gaussian foreign local factor, $Y_4(t)$. The two nominal interest rates are given as:

$$r_d(t) = \delta_0^d + \delta_1^d Y_1(t) + \delta_2^d Y_2(t) + \delta_3^d Y_3(t), \tag{39}$$

$$r_f(t) = \delta_0^f + \delta_1^f Y_1(t) + \delta_2^f Y_2(t) + \delta_4^f Y_4(t). \tag{40}$$

The SDEs for the state variables are

$$d\begin{pmatrix} Y_1(t) \\ Y_2(t) \\ Y_3(t) \\ Y_4(t) \end{pmatrix} = \begin{pmatrix} \kappa_{11} & \kappa_{12} & 0 & 0 \\ \kappa_{21} & \kappa_{22} & 0 & 0 \\ \kappa_{31} & \kappa_{32} & \kappa_{33} & 0 \\ \kappa_{41} & \kappa_{42} & 0 & \kappa_{44} \end{pmatrix} \begin{pmatrix} \theta_1 & - & Y_1(t) \\ \theta_2 & - & Y_2(t) \\ & - & Y_3(t) \\ & - & Y_4(t) \end{pmatrix} dt$$
$$+ \begin{pmatrix} \sqrt{Y_1(t)} & 0 & 0 & 0 \\ 0 & \sqrt{Y_2(t)} & 0 & 0 \\ 0 & 0 & \sqrt{1+\beta_{31}Y_1(t)+\beta_{32}Y_2(t)} & \\ 0 & 0 & 0 & \sqrt{1+\beta_{41}Y_1(t)+\beta_{42}Y_2(t)} \end{pmatrix} dW(t) \tag{41}$$

By Ito's lemma, the SDEs of the interests are written as:

$$\begin{aligned} dr_d(t) &= \left[\sum_{i=1}^{3} \delta_i^d \kappa_{i1}(\theta_1 - Y_1(t)) - \sum_{i=1}^{3} \delta_i^d \kappa_{i2}(\theta_2 - Y_2(t)) - \delta_3^d \kappa_3 3 Y_3(t)\right] dt \\ &+ \delta_1^d \sqrt{Y_1(t)} dW_1(t) + \delta_2^d \sqrt{Y_2(t)} dW_2(t) + \delta_3^d \sqrt{1+\beta_{31}Y_1(t)+\beta_{32}Y_2(t)} dW_3(t), \end{aligned} \tag{42}$$

$$\begin{aligned} dr_f(t) &= \left[\sum_{i=1;i\neq 3}^{4} \delta_i^f \kappa_{i1}(\theta_1 - Y_1(t)) - \sum_{i=1;i\neq 3}^{4} \delta_i^f \kappa_{i2}(\theta_2 - Y_2(t)) - \delta_3^f \kappa_4 4 Y_4(t)\right] dt \\ &+ \delta_1^f \sqrt{Y_1(t)} dW_1(t) + \delta_2^f \sqrt{Y_2(t)} dW_2(t) + \delta_4^f \sqrt{1+\beta_{41}Y_1(t)+\beta_{42}Y_2(t)} dW_3(t). \end{aligned} \tag{43}$$

The instantaneous correlation of the two-country interest rates becomes

$$\mathrm{Corr}_{df}(t) = \frac{\mathrm{Cov}_{df}(t)}{\sqrt{\mathrm{Var}_d(t)}\sqrt{\mathrm{Var}_f(t)}}, \tag{44}$$

where

$$\begin{aligned} \mathrm{Var}_d(t) &= \delta_1^{d^2} Y_1(t) + \delta_2^{d^2} Y_2(t) + \delta_3^{d^2}\left(1+\beta_{31}Y_1(t)+\beta_{32}Y_2(t)\right), \\ \mathrm{Var}_f(t) &= \delta_1^{f^2} Y_1(t) + \delta_2^{f^2} Y_2(t) + \delta_4^{f^2}\left(1+\beta_{41}Y_1(t)+\beta_{42}Y_2(t)\right), \\ \mathrm{Cov}_{df}(t) &= \delta_1^d \delta_1^f Y_1(t) + \delta_2^d \delta_2^f Y_2(t). \end{aligned}$$

Equation (44) indicates that the correlation is governed by the two square-root common factors, $Y_1(t)$ and $Y_2(t)$, which implies that this model can generate the sign-switching correlation through either $\delta_1^d \delta_1^f < 0$ or $\delta_2^d \delta_2^f < 0$.

$\mathbb{IA}_{3;1,1,1}(4;2,1,1)$

This model includes three square-root factors, $Y_1(t), Y_2(t)$, and $Y_3(t)$, and one Gaussian factor, $Y_4(t)$, wherein $Y_1(t)$ and $Y_4(t)$ are common factors, and $Y_2(t)$ and $Y_3(t)$ are domestic and foreign local factors, respectively. The relationship among the state variables and the interest rate in each local market is given as:

$$r_d(t) = \delta_0^d + \delta_1^d Y_1(t) + \delta_2^d Y_2(t) + \delta_4^d Y_4(t), \tag{45}$$

$$r_f(t) = \delta_0^f + \delta_1^f Y_1(t) + \delta_3^f Y_3(t) + \delta_4^f Y_4(t). \tag{46}$$

The state variables are assumed to follow the SDEs:

$$\begin{aligned} d\begin{pmatrix} Y_1(t) \\ Y_2(t) \\ Y_3(t) \\ Y_4(t) \end{pmatrix} &= \begin{pmatrix} \kappa_{11} & 0 & 0 & 0 \\ \kappa_{21} & \kappa_{22} & 0 & 0 \\ \kappa_{31} & 0 & \kappa_{33} & 0 \\ \kappa_{41} & 0 & 0 & \kappa_{44} \end{pmatrix} \begin{pmatrix} \theta_1 & - & Y_1(t) \\ \theta_2 & - & Y_2(t) \\ \theta_3 & - & Y_3(t) \\ & - & Y_4(t) \end{pmatrix} dt \\ &+ \begin{pmatrix} \sqrt{Y_1(t)} & 0 & 0 & 0 \\ 0 & \sqrt{Y_2(t)} & 0 & 0 \\ 0 & 0 & \sqrt{Y_3(t)} & \\ 0 & 0 & 0 & \sqrt{1+\beta_{41}Y_1(t)} \end{pmatrix} dW(t). \end{aligned} \tag{47}$$

By Ito's lemma, the SDEs of the interest becomes

$$\begin{aligned} dr_d(t) &= \left[\sum_{i=1;i\neq 3}^{4} \delta_i^d \kappa_{i1} \left(\theta_1 - Y_1(t)\right) - \delta_2^d \kappa_{22} \left(\theta_2 - Y_2(t)\right) - \delta_4^d \kappa_4 4 Y_4(t) \right] dt \\ &+ \delta_1^d \sqrt{Y_1(t)} dW_1(t) + \delta_2^d \sqrt{Y_2(t)} dW_2(t) + \delta_4^d \sqrt{1+\beta_{41}Y_1(t))} dW_4(t), \end{aligned} \tag{48}$$

$$\begin{aligned} dr_f(t) &= \left[\sum_{i=1;i\neq 2}^{4} \delta_i^f \kappa_{i1} \left(\theta_1 - Y_1(t)\right) - \delta_3^f \kappa_{33} \left(\theta_3 - Y_3(t)\right) - \delta_4^f \kappa_4 4 Y_4(t) \right] dt \\ &+ \delta_1^f \sqrt{Y_1(t)} dW_1(t) + \delta_3^f \sqrt{Y_3(t)} dW_3(t) + \delta_4^f \sqrt{1+\beta_{41}Y_1(t))} dW_4(t), \end{aligned} \tag{49}$$

The correlation coefficient of the two interest rates can be represented as:

$$\text{Corr}_{df}(t) = \frac{\text{Cov}_{df}(t)}{\sqrt{\text{Var}_d(t)}\sqrt{\text{Var}_f(t)}}, \tag{50}$$

where

$$\begin{aligned} \text{Var}_d(t) &= \delta_1^{d^2} Y_1(t) + \delta_2^{d^2} Y_2(t) + \delta_4^{d^2} \left(1 + \beta_{41} Y_1(t)\right), \\ \text{Var}_f(t) &= \delta_1^{f^2} Y_1(t) + \delta_3^{f^2} Y_3(t) + \delta_4^{f^2} \left(1 + \beta_{41} Y_1(t)\right), \\ \text{Cov}_{df}(t) &= \delta_1^d \delta_1^f Y_1(t) + \delta_4^d \delta_4^f \left(1 + \beta_{41} Y_1(t)\right). \end{aligned}$$

Therefore the sign-switching property of the correlation is governed by the square-root common factor, $Y_1(t)$. This model can accommodate the sign-switching correlation through either $\delta_1^d \delta_1^f < 0$ or $\delta_4^d \delta_4^f < 0$.

3.5 Comparative characteristics of the models

In this subsection, we highlight some of the similarities and differences among the $\mathbb{IATSM}$s considered in this paper. First, none of the models in Section 3.4 are able to guarantee the positivity of the nominal interest rates, $r_d(t)$ and $r_f(t)$, because there is one or more Gaussian factors affecting the processes of $r_d(t)$ and $r_f(t)$ for all models. Second, considering that the instantaneous variances of the two-country interest rates and bond prices are determined by the square-root factors, $\mathbb{IA}_{1;1,0,0}(3;3,0,0)$ shows least flexibility in inducing heteroskedastic volatility because it has only one square-root common factor, $Y_1(t)$. In both $\mathbb{IA}_{2;2,0,0}(3;3,0,0)$ and $\mathbb{IA}_{2;2,0,0}(4;2,1,1)$, conditional heteroskedasticity is driven by the two square-root common factors, $Y_1(t)$ and $Y_2(t)$. Similarly, two square-root factors generate heteroskedastic volatility in $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$. However, $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$ includes the square-root local factors, $Y_2(t)$ and $Y_3(t)$, as well as the square-root common factor, $Y_1(t)$, which states that the stochastic volatility of $r_d(t)$ ($r_f(t)$) is driven by the common factor, $Y_1(t)$, and the domestic (foreign) local factor, $Y_2(t)$ ($Y_3(t)$). Thus, unlike other models, $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$ is able to accommodate the time-varying component in volatility dynamics that are not linked to the covariance. Third, $\mathbb{IA}_{1;1,0,0}(3;3,0,0)$ is most flexible in specifying the correlation structures among the state variables. This model has only one square-root factor, and, therefore, all factors can be negatively correlated with each other. $\mathbb{IA}_{2;2,0,0}(3;3,0,0)$ is isomorphic to $\mathbb{IA}_{2;2,0,0}(4;2,1,1)$ in allowing negative correlations among factors because each local market is represented as $A_2(3)$ in both models. The factor structure of $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$ limits its flexibility in specifying negative correlations among the state variables. In $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$, the zero restrictions on κ_{42} and κ_{43} indicates that the Gaussian common factor, $Y_4(t)$, cannot be correlated with the two square-root local factors, $Y_2(t)$ and $Y_3(t)$. Otherwise each local factor can affect the process of the common factor $Y_4(t)$.[18] Fourth, as an increase in the number, m_c, of the square-root common factors gives more flexibility to the $\mathbb{IATSM}$s in their modeling of the sign-switching correlation of the interest rates, $\mathbb{IA}_{2;2,0,0}(3;3,0,0)$ and $\mathbb{IA}_{2;2,0,0}(4;2,1,1)$ are more flexible than models $\mathbb{IA}_{1;1,0,0}(3;3,0,0)$ and $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$ in generating the sign-switching property of the correlation. In the two former models, two square-root common factors, as opposed to a single square-root factor, can contribute to generating the sign-switching correlation. Fifth, both $\mathbb{IA}_{1;1,0,0}(3;3,0,0)$ and $\mathbb{IA}_{2;2,0,0}(3;3,0,0)$ are incapable of capturing the country-specific movements of the interest rates by construction. In contrast, $\mathbb{IA}_{2;2,0,0}(4;2,1,1)$ can accommodate local dynamics through the two local factors, $Y_3(t)$ and $Y_4(t)$. In the case of $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$, $Y_2(t)$ and $Y_3(t)$ capture the country-specific movements of the interest rates and bond prices of the two countries.

4 A Characterization of the International Quadratic Term Structure Models

4.1 A canonical form of $\mathbb{IQTSM}$s

This section presents a canonical model of $\mathbb{IQTSM}$, which is a two-country extension of the single-country $\mathbb{QTSM}$s of Ahn, Dittmar, and Gallant (2002).

An N-factor $\mathbb{IQTSM}$ can be completely specified by the following three assumptions. First, the

[18] In $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$, the two local factors, $Y_2(t)$ and $Y_3(t)$, and the common factor, $Y_4(t)$, cannot be correlated in such a way of the effects of $Y_4(t)$ on the mean dynamics of $Y_2(t)$ and $Y_3(t)$ (i.e., κ_{24} and κ_{34}) because of the admissibility conditions provided by Dai and Singleton (2000).

instantaneous interest rate of country k is a quadratic function of the state variables:

$$r_k(t) = \alpha^k + \beta^{k\prime} Y(t) + Y(t)' \Psi^k Y(t), \tag{51}$$

where α^k is a constant, β^k is an $N \times 1$ vector, and Ψ^k is an $N \times N$ matrix. Second, the SDEs of the state variables are given as:

$$dY(t) = [\Phi_0 + \Phi_1 Y(t)] + \Sigma dW(t), \tag{52}$$

where Φ_0 is an $N \times 1$ vector, and Φ_1 and Σ are $N \times N$ matrices. As such, the time series process of the state variables is represented as a Gaussian process. Third, the diffusion term of the global SDF denominated in currency k is represented as an affine function of the state variables:

$$\Lambda_k(t) = \eta_0^k + \eta_1^k Y(t), \tag{53}$$

where η_0^k is a $N \times 1$ vector, and η_1^k an N-dimensional matrix.

We classify N-factor $\mathbb{IQTSM}$s by the number of common, domestic local, and foreign local factors, N_c, N_d, and N_f. Let $\mathbb{IQ}(N; N_c, N_d, N_f)$ denote an N-factor $\mathbb{IQTSM}$ that is empirically identifiable. Our canonical representation of $\mathbb{IQ}(N; N_c, N_d, N_f)$ is obtained by imposing the factor structure of Assumption 1 on the canonical model of the single-country $\mathbb{QTSM}$s of Ahn, Dittmar, and Gallant (2002).

Definition 2: *Partitioning Y as*

$$Y' = \left({Y^c_{N_c \times 1}}', {Y^d_{N_d \times 1}}', {Y^f_{N_f \times 1}}' \right),$$

and W as

$$W' = \left({W^c_{N_c \times 1}}', {W^d_{N_d \times 1}}', {W^f_{N_f \times 1}}' \right),$$

we define the canonical representation of $\mathbb{IQ}(N; N_c, N_d, N_f)$ by adding the following restrictions on equations (51) and (52):

$$\beta^{d\prime} = \left(0_{N_c \times 1}', 0_{N_d \times 1}', 0_{N_f \times 1}' \right), \tag{54}$$

$$\beta^{f\prime} = \left({\beta^f_{c\,N_c \times 1}}', 0_{N_d \times 1}', 0_{N_f \times 1}' \right), \tag{55}$$

$$\Psi^d = \begin{bmatrix} \Psi^d_{cc(N_c \times N_c)} & \Psi^d_{cd(N_c \times N_d)} & 0_{(N_c \times N_f)} \\ \Psi^{d\,\prime}_{cd(N_d \times N_c)} & \Psi^d_{dd(N_d \times N_d)} & 0_{(N_d \times N_f)} \\ 0_{(N_f \times N_c)} & 0_{(N_f \times N_d)} & 0_{(N_f \times N_f)} \end{bmatrix}, \tag{56}$$

$$\Psi^f = \begin{bmatrix} \Psi^f_{cc(N_c \times N_c)} & 0_{(N_c \times N_d)} & \Psi^f_{cf\,(N_c \times N_f)} \\ 0_{(N_d \times N_c)} & 0_{(N_d \times N_d)} & 0_{(N_d \times N_f)} \\ \Psi^{f\,\prime}_{cf\,(N_f \times N_c)} & 0_{(N_f \times N_d)} & \Psi^f_{ff\,(N_f \times N_f)} \end{bmatrix}, \tag{57}$$

where Ψ^d_{cc}, Ψ^d_{dd}, and Ψ^f_{ff} are symmetric matrices with diagonal terms of 1s, and Ψ^f_{cc} is a symmetric matrix,

$$\Phi_0' = \left({\Phi^c_{0(N_c \times 1)}}', {\Phi^d_{0(N_d \times 1)}}', {\Phi^f_{0(N_f \times 1)}}' \right), \tag{58}$$

where Φ^c_0, Φ^d_0, *and* Φ^f_0 *are vectors of non-negative constants,*

$$\Phi_1 = \begin{bmatrix} \Phi^{cc}_{1\,(N_c \times N_c)} & 0_{(N_c \times N_d)} & 0_{(N_c \times N_f)} \\ \Phi^{dc}_{1\,(N_d \times N_c)} & \Phi^{dd}_{1\,(N_d \times N_d)} & 0_{(N_d \times N_f)} \\ \Phi^{fc}_{1\,(N_f \times N_c)} & 0_{(N_f \times N_d)} & \Phi^{ff}_{1\,(N_f \times N_f)} \end{bmatrix}, \tag{59}$$

where Φ^{cc}_1, Φ^{dd}_1, *and* Φ^{ff}_1 *are lower triangular matrices,*

$$\Sigma = \begin{bmatrix} \Sigma^{cc}_{(N_c \times N_c)} & 0_{(N_c \times N_d)} & 0_{(N_c \times N_f)} \\ 0_{(N_d \times N_c)} & \Sigma^{dd}_{(N_d \times N_d)} & 0_{(N_d \times N_f)} \\ 0_{(N_f \times N_c)} & 0_{(N_f \times N_d)} & \Sigma^{ff}_{(N_f \times N_f)} \end{bmatrix}, \tag{60}$$

where Σ^{cc}, Σ^{dd}, *and* Σ^{ff} *are diagonal matrices.*

In our canonical representation, the process of the domestic interest rate, $r_d(t)$, is determined by the common factors, $Y^c(t)$, and the domestic local factors, $Y^d(t)$. Similarly, we assume that the dynamics of the foreign interest rate, $r_f(t)$, is driven by the common and foreign local factors, $Y^c(t)$ and $Y^f(t)$. From equations (54) and (56), the domestic interest rate is not influenced by the foreign local factors. Equations (55) and (57) ensure that the domestic local factors cannot affect the foreign interest rate. The specifications of the drift term, Φ_0 and Φ_1, and the diffusion term, Σ, correspond to the structure of the state variables in Assumption 1, which ensures that the domestic (foreign) local factors cannot indirectly affect the foreign (domestic) interest rate.[19]

4.2 An ability of $\mathbb{IQTSM}$s to generate the sign-switching correlations

We turn next to the characteristics of $\mathbb{IQTSM}$s in generating the sign-switching correlation of the two interest rates, $r_d(t)$ and $r_f(t)$. From Ito's lemma, the instantaneous covariance of $r_d(t)$ and $r_f(t)$ can be represented as:

$$\mathrm{Cov}_{df}(t) = 4 \left[Y^c(t)' \Psi^d_{cc} + Y^d(t)' {\Psi^d_{cd}}' \right] \Sigma^{cc} \Sigma^{cc\prime} \left[\Psi^f_{cf} Y^f(t) + \Psi^f_{cc} Y^c(t) + \frac{1}{2} \beta^f_c \right]. \tag{61}$$

Equation (61) states that the covariance is a quadratic function of the common, domestic local, and foreign local factors, $Y^c(t)$, $Y^d(t)$, and $Y^f(t)$. Unlike $\mathbb{IATSM}$s, both the domestic and foreign local factors affect the covariance. As such, all included factors can contribute to generating the sign-switching correlation of the interest rates. This property arises from the quadratic relationship between the state variables and the short rates. Remember that the covariance of $r_d(t)$ and $r_f(t)$ induced by $\mathbb{IATSM}$s is represented as an affine function of the square-root common

[19]Similar to $\mathbb{IATSM}$s, the identifiability conditions for $\mathbb{IQTSM}$s are more flexible than those of the single-country $\mathbb{QTSM}$s. We assume that $\left[\beta^d\right]_i = 0, 1 \leq i \leq N_c$. Identifying the level of the common factors, $Y^c(t)$, in this way allows Φ^c_0 and β^f_c to be treated as free parameters. In order to fix the scale of $Y^c(t)$, we assume that the diagonal terms of Ψ^d_{cc} are 1. These N_c restrictions allow us to simultaneously identify the diagonal terms of Ψ^f_{cc}, Φ^{cc}_1, and Σ^{cc}. The remaining identifiability conditions are the same as those of the single-country $\mathbb{QTSM}$s provided by Ahn, Dittmar, and Gallant (2002).

factors, $Y^{B_c}(t)$, only. In equation (61), given the indefiniteness of matrix $\Psi^d_{cc}\Sigma^{cc}\Sigma^{cc\prime}\Psi^f_{cc}$ and the existence of the bilinear term of $Y^d(t)$ and $Y^f(t)$, $Y^d(t)'\Psi^{d\,\prime}_{cd}\Sigma^{cc}\Sigma^{cc\prime}\Psi^f_{cf}Y^f(t)$, and the linear term of $Y^d(t)$, $\frac{1}{2}Y^d(t)'\Psi^{d\,\prime}_{cd}\Sigma^{cc}\Sigma^{cc\prime}\beta^f_c$, the covariance can stochastically change sign over time. In the following subsection, we investigate in detail this form of covariance by focusing on a three-factor $\mathbb{IQTSM}$, wherein we show that the sign-switching correlation of the interest rates can be theoretically generated from the Gaussian property of the state variables.

From our canonical form, we can establish the following proposition regarding the characteristics of $\mathbb{IQTSM}$s.

Proposition 4: *A subfamily of three-factor $\mathbb{IQ}(N; N_c, N_d, N_f)$ with $N_c = N_d = N_f = 1$ can simultaneously (i) guarantee the positivity of $r_d(t)$ and $r_f(t)$, (ii) allow for the negative correlations among the state variables, (iii) generate the heteroskedastic volatility of $r_d(t)$ and $r_f(t)$, (iv) induce the sign-switching correlation of $r_d(t)$ and $r_f(t)$, and (v) capture the country-specific movements of $r_d(t)$ and $r_f(t)$.*
Proof. See section 4.3.

This proposition and the form of covariance in equation (61) highlight important differences between $\mathbb{ATSM}$s and $\mathbb{QTSM}$s when they are extended to a two-country setup. First, $\mathbb{IQTSM}$s are able to generate the sign-switching correlation of the two interest rates, $r_d(t)$ and $r_f(t)$, without violating the positivity of the interest rates. However, as demonstrated in Proposition 1, there is no $\mathbb{IATSM}$ that can simultaneously guarantee the positivity of the interest rates and accommodate the sign-switching cross-country interest rate correlation. Second, a subfamily of $\mathbb{IQTSM}$s including three factors can accommodate both the sign-switching correlation and country-specific movements of the interest rates. However, as shown in Proposition 3, three-factor $\mathbb{IATSM}$s can generate the sign-switching correlation by giving up the ability to capture the country-specific dynamics. Conversely, they are able to accommodate the country-specific dynamics at the cost of matching the sign-switching correlation. Therefore, $\mathbb{IQTSM}$s are potentially more efficient than $\mathbb{IATSM}$s in the usage of the state variables. Third, the forms of the covariance of $r_d(t)$ and $r_f(t)$ induced by $\mathbb{IATSM}$s and $\mathbb{IQTSM}$s state that $\mathbb{IQTSM}$s are theoretically free of the trade-off between the flexibility in generating the sign-switching property of the correlation of the interest rates and in accommodating the negative correlations among the factors, which are observed in $\mathbb{IATSM}$s. An increase in the number of square-root common factors limits the flexibility of an $\mathbb{IATSM}$ in specifying conditional/unconditional correlations among the state variables while giving more flexibility in specifying the sign-switching correlation of the interest rates. In contrast, in $\mathbb{IQTSM}$s, all included factors can contribute to generating the sign-switching correlation of the interest rates. Due to their Gaussian property, all of the state variables can be negatively correlated.

To complete the valuation of bonds, we assume that the diffusion term of the global SDF defined on currency k is given as:

$$\Lambda_k(t) = \begin{pmatrix} \eta^k_{0,c} \\ \eta^k_{0,d} \\ \eta^k_{0,f} \end{pmatrix} + \begin{pmatrix} \eta^k_{1,cc} & 0 & 0 \\ \eta^k_{1,dc} & \eta^k_{1,dd} & 0 \\ \eta^k_{1,fc} & 0 & \eta^k_{1,ff} \end{pmatrix} \begin{pmatrix} Y^c(t) \\ Y^d(t) \\ Y^f(t) \end{pmatrix}. \tag{62}$$

Based on these assumptions, we can write the price of a discount bond as an exponential quadratic function of the state vector:

$$P_k(t,\tau) = exp\left[A^k(\tau) + B^k(\tau)'Y(t) + Y(t)'C^k(\tau)Y(t)\right], \tag{63}$$

where

$$B^d(\tau)' = \left(B^d_c(\tau)', B^d_d(\tau)', 0'\right), \; B^f(\tau)' = \left(B^f_c(\tau)', 0', B^f_f(\tau)'\right),$$

$$C^d(\tau) = \begin{pmatrix} C^d_{cc}(\tau) & C^d_{cd}(\tau) & 0 \\ C^d_{dc}(\tau) & C^d_{dd}(\tau) & 0 \\ 0 & 0 & 0 \end{pmatrix}, \; C^f(\tau) = \begin{pmatrix} C^f_{cc}(\tau) & 0 & C^f_{cf}(\tau) \\ 0 & 0 & 0 \\ C^f_{fc}(\tau) & 0 & C^f_{ff}(\tau) \end{pmatrix}.$$

As shown by Ahn, Dittmar, and Gallant (2002), $A^k(\tau)$, $B^k(t,\tau)$, and $C^k(t,\tau)$ satisfy the ODEs:

$$\begin{aligned} \frac{dC^k(\tau)}{d\tau} &= 2C^k(\tau)\Sigma\Sigma'C^k(\tau) + \left[C^k(\tau)(\Phi_1 - \Sigma\eta^k_1) + (\Phi_1 - \Sigma\eta^k_1)'C^k(\tau)\right] - \Psi^k, \\ \frac{dB^k(\tau)}{d\tau} &= 2C^k(\tau)\Sigma\Sigma'B^k(\tau) + (\Phi_1 - \Sigma\eta^k_1)'B^k(\tau) + 2C^k(\tau)(\Phi_0 - \Sigma\eta^k_0) - \beta^k, \\ \frac{dA^k(\tau)}{d\tau} &= \mathrm{tr}[\Sigma\Sigma'C^k(\tau)] + \frac{1}{2}B^k(\tau)'\Sigma\Sigma'B^k(\tau) + B^k(\tau)'(\Phi_0 - \Sigma\eta^k_0) - \alpha^k, \end{aligned}$$

with the initial conditions $A^k(0) = 0_{1\times1}$, $B^k(0) = 0_{N\times1}$, and $C^k(0) = 0_{N\times N}$. The yield-to-maturity $yt_k(t,\tau)$ is defined as $-\ln P_k(t,\tau)/\tau$:

$$yt_k(t,\tau) = \frac{1}{\tau}\left[-\ln A^k(\tau) - B^k(\tau)'Y(t) - Y(t)'C^k(\tau)'Y(t)\right].$$

Thus the domestic bond yield, $yt_d(t,\tau)$, is a quadratic function of the common and domestic local factors, $Y^f(t)$ and $Y^d(t)$. Similarly, the foreign bond yield, $yt_f(t,\tau)$, is a quadratic function of the common factors , $Y^c(t)$, and the foreign local factors, $Y^f(t)$.

4.3 Three factor $\mathbb{IQTSM}$s

This subsection presents a three-factor $\mathbb{IQTSM}$ considered in this paper. There are two sub-families of three-factor $\mathbb{IQTSM}$s: $\mathbb{IQ}(3;3,0,0)$ and $\mathbb{IQ}(3;1,1,1)$. Among these models, we prefer $\mathbb{IQ}(3;1,1,1)$ because this model includes not only a common factor but also two local factors, while $\mathbb{IQ}(3;3,0,0)$ includes common factors only. In $\mathbb{IQ}(3;1,1,1)$, the dynamics of the two interest rates, $r_d(t)$ and $r_f(t)$, and bond yields in each local market are described by a two factor single-country $\mathbb{QTSM}$. In contrast, all the three-and four-factor $\mathbb{IATSM}$s considered in this paper collapse to three-factor single-country $\mathbb{ATSM}$s.

$\underline{\mathbb{IQ}(3;1,1,1)}$

This model designates the domestic interest rate, $r_d(t)$, as a quadratic function of common factor, $Y_1(t)$, and domestic local factor, $Y_2(t)$. The foreign interest rate, $r_f(t)$, is assumed to be a quadratic function of common factor, $Y_1(t)$, and foreign local factor, $Y_3(t)$. Specifically, we assume that

$$r_d(t) = \alpha^d + Y(t)'\Psi^d Y(t), \tag{64}$$

where

$$\Psi^d = \begin{pmatrix} 1 & \Psi_{12}^d & 0 \\ \Psi_{12}^d & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

and

$$r_f(t) = \alpha^f + \beta^{f\prime} Y(t) + Y(t)' \Psi^f Y(t), \tag{65}$$

where

$$\beta^f = \left(\beta_1^f, 0, 0\right)',$$

$$\Psi^f = \begin{pmatrix} \Psi_{11}^f & 0 & \Psi_{13}^f \\ 0 & 0 & 0 \\ \Psi_{13}^f & 0 & 1 \end{pmatrix}.$$

One of the advantages of $\mathbb{IQTSM}$s over $\mathbb{IATSM}$s is that $\mathbb{IQTSM}$s are able to guarantee the positivity of the two nominal interest rates, $r_d(t)$ and $r_f(t)$; $\mathbb{IATSM}$s cannot, in general, guarantee this desirable property. By assuming that $\alpha^d > 0$, and Ψ^d is a positive semidefinite matrix, we ensure the positivity of the domestic interest rate. From the assumption that Ψ^f is positive semidefinite, the lower bound on the foreign interest rate can be written as $\alpha^f - \frac{1}{4}\beta^{f\prime}\Psi^{f^{-1}}\beta^f$. Thus the non-negativity of the foreign interest rate can be assured if the sign of its lower bound is non-negative.

The SDEs of the state variables are given as:

$$\begin{aligned} d\begin{pmatrix} Y_1(t) \\ Y_2(t) \\ Y_3(t) \end{pmatrix} &= \left[\begin{pmatrix} \Phi_{01} \\ \Phi_{02} \\ \Phi_{03} \end{pmatrix} + \begin{pmatrix} \Phi_{111} & 0 & 0 \\ \Phi_{121} & \Phi_{122} & 0 \\ \Phi_{131} & 0 & \Phi_{133} \end{pmatrix} \begin{pmatrix} Y_1(t) \\ Y_2(t) \\ Y_3(t) \end{pmatrix}\right] dt \\ &+ \begin{pmatrix} \Sigma_{11} & 0 & 0 \\ 0 & \Sigma_{22} & 0 \\ 0 & 0 & \Sigma_{33} \end{pmatrix} dW(t). \end{aligned} \tag{66}$$

In equation (66), the signs of the off-diagonal terms of Φ_1 are unconstrained, and, therefore, correlations among the state variables may be positive or negative. Φ_{121} (Φ_{131}) can induce negative correlation between the common factor, $Y_1(t)$, and the domestic (foreign) local factor, $Y_2(t)$ ($Y_3(t)$).

Applying Ito's lemma to equations (64) and (65) leads to the following SDEs of the two interest rates:

$$\begin{aligned} dr_d(t) &= \Big[\Sigma_{11}^2 + \Sigma_{22}^2 + 2\left(\Phi_{01} + \Phi_{111}Y_1(t)\right)\left(Y_1(t) + \Psi_{12}^d Y_2(t)\right) \\ &+ 2\left(\Phi_{02} + \Phi_{121}Y_1(t) + \Phi_{122}Y_2(t)\right)\left(\Psi_{12}^d Y_1(t) + Y_2(t)\right)\Big] dt \\ &+ 2\Sigma_{11}\left(Y_1(t) + \Psi_{12}^d Y_2(t)\right) dW_1(t) + 2\Sigma_{22}\left(\Psi_{12}^d Y_1(t) + Y_2(t)\right) dW_2(t), \end{aligned} \tag{67}$$

$$dr_f(t) = \Big[\Psi_{11}^f \Sigma_{11}^2 + \Sigma_{33}^2 + 2\left(\Phi_{01} + \Phi_{111}Y_1(t)\right)\left(\frac{\beta_1^f}{2} + \Psi_{11}^f Y_1(t) + \Psi_{13}^f Y_3(t)\right)$$

$$+2\left(\Phi_{03} + \Phi_{131} Y_1(t) + \Phi_{133} Y_3(t)\right)\left(\Psi^f_{13} Y_1(t) + Y_3(t)\right)\Big] dt$$

$$+2\Sigma_{11}\left(\frac{\beta^f_1}{2} + \Psi^f_{11} Y_1(t) + \Psi^f_{13} Y_3(t)\right) dW_1(t) + 2\Sigma_{33}\left(\Psi^f_{13} Y_1(t) + Y_3(t)\right) dW_3(t) \quad (68)$$

Then the instantaneous variances of the interest rate changes are represented as:

$$\mathrm{Var}_d(t) = 4\Sigma^2_{11}\left(Y_1(t) + \Psi^d_{12} Y_2(t)\right)^2 + 4\Sigma^2_{22}\left(\Psi^d_{12} Y_1(t) + Y_2(t)\right)^2, \quad (69)$$

$$\mathrm{Var}_f(t) = 4\Sigma^2_{11}\left(\frac{\beta^f_1}{2} + \Psi^f_{11} Y_1(t) + \Psi^f_{13} Y_3(t)\right)^2 + 4\Sigma^2_{33}\left(\Psi^f_{13} Y_1(t) + Y_3(t)\right)^2. \quad (70)$$

Thus all relevant factors can contribute to generating the heteroskedastic volatility of $r_d(t)$ and $r_f(t)$.

We turn next to the mechanism for generating the sign-switching correlation. The covariance of the two interest rates can be written as:

$$\mathrm{Cov}_{df}(t) = 4\Sigma^2_{11}\left(Y_1(t) + \Psi^d_{12} Y_2(t)\right)\left(\frac{1}{2}\beta^f_1 + \Psi^f_{11} Y_1(t) + \Psi^f_{13} Y_3(t)\right). \quad (71)$$

Thus the instantaneous covariance of the interest rates is a quadratic function of all included state variables. Designating each short rate as a quadratic function of the state variables, which guarantees the positivity of the interest rates, allows the two local factors, $Y_2(t)$ and $Y_3(t)$, to influence the covariance. Rearranging the terms of equation (71) yields the following expression

$$\begin{aligned}\mathrm{Cov}_{df}(t) = & \left[2\Sigma_{11}\sqrt{\Psi^f_{11}} Y_1(t) + \frac{\Sigma_{11}}{\sqrt{\Psi^f_{11}}}\left(\frac{1}{2}\beta^f_1 + \Psi^f_{11}\Psi^d_{12} Y_2(t) + \Psi^f_{13} Y_3(t)\right)\right]^2 \\ & - \left[\frac{\Sigma_{11}}{\sqrt{\Psi^f_{11}}}\left(\frac{1}{2}\beta^f_1 + \Psi^f_{11}\Psi^d_{12} Y_2(t) + \Psi^f_{13} Y_3(t)\right)\right]^2 \\ & + \frac{1}{2}\beta^f_1 \Psi^d_{12} Y_2(t) + \Psi^d_{12}\Psi^f_{13} Y_2(t) Y_3(t). \end{aligned} \quad (72)$$

Equation (72) states the covariance can be viewed as composed of three parts: the terms in the first two square brackets (quadratic term) and the remaining two terms (linear and bilinear terms). As such, there are three channels through which the covariance can switch sign over time. First, conditioning on the two local factors, $Y_2(t)$ and $Y_3(t)$, the quadratic term attains its lower bound, $-\frac{\Sigma^2_{11}}{\Psi^f_{11}}\left(\frac{1}{2}\beta^f_1 + \Psi^f_{11}\Psi^d_{12} Y_2(t) + \Psi^f_{13} Y_3(t)\right)]^2$, which is strictly negative. Therefore, the quadratic term takes negative values when the sample path of $Y_1(t)$ remains between the two intercepts: zero and $-\frac{1}{\Psi^f_{11}}\left(\frac{1}{2}\beta^f_1 + \Psi^f_{11}\Psi^d_{12} Y_2(t) + \Psi^f_{13} Y_3(t)\right)$. Otherwise, it takes positive values. Second, the linear and bilinear terms can stochastically switch sign over time from the Gaussian property of $Y_2(t)$ and $Y_3(t)$.

4.4 A comparison of the mechanism for generating sign-switching correlation of interest rates

We conclude this section by presenting a comparison of the sign-switching correlation generating mechanism of $\mathbb{IATSM}$s and $\mathbb{IQTSM}$s by focusing on the models to be estimated in this paper.

For this purpose, we classify the four $\mathbb{IATSM}$s by the number, m_c, of the common square-root factors, $Y^{B_c}(t)$. In both $\mathbb{IA}_{1;1,0,0}(3;3,0,0)$ and $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$, only one common square-root factor drives the instantaneous covariance of the interest rates, while two square-root factors can contribute to generating the sign-switching property of the correlation for both $\mathbb{IA}_{2;2,0,0}(3;3,0,0)$ and $\mathbb{IA}_{2;2,0,0}(4;2,1,1)$.

Comparing $\mathbb{IA}_{1;1,0,0}(3;3,0,0)$ and $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$, with $\mathbb{IQ}(3;1,1,1)$

As discussed in the previous section, in both $\mathbb{IA}_{1;1,0,0}(3;3,0,0)$ and $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$, the covariance of the two interest rates, $r_d(t)$ and $r_f(t)$, is represented as an affine function of the square-root common factor, $Y_1(t)$. To be able to induce the sign-switching property of the correlation between $r_d(t)$ and $r_f(t)$, both models require some parametric restrictions. In the case of $\mathbb{IA}_{1;1,0,0}(3;3,0,0)$, one or two terms of $(\delta_1^d\delta_1^f, \delta_2^d\delta_2^f, \delta_3^d\delta_3^f)$ must take negative value. $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$ requires that either $\delta_1^d\delta_1^f < 0$ or $\delta_4^d\delta_4^f < 0$. These two affine models can induce the sign-switching property of the correlation when the sample path of $Y_1(t)$ passes the intercept of the covariance function of $r_d(t)$ and $r_f(t)$: $-\frac{\delta_2^d\delta_2^f+\delta_3^d\delta_3^f}{\delta_1^d\delta_1^f+\delta_2^d\delta_2^f\beta_{21}+\delta_3^d\delta_3^f\beta_{31}}$ for $\mathbb{IA}_{1;1,0,0}(3;3,0,0)$, and $-\frac{\delta_4^d\delta_4^f}{\delta_1^d\delta_1^f+\delta_4^d\delta_4^f\beta_{41}}$ for $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$. This mechanism can potentially impose a restriction on the admissible range of $Y_1(t)$ for both models. Among these models, $\mathbb{IA}_{1;1,0,0}(3;3,0,0)$ is more restrictive than $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$. As is represented in equation (34), the instantaneous variances of $r_d(t)$ and $r_f(t)$ are affine functions of $Y_1(t)$ only in $\mathbb{IA}_{1;1,0,0}(3;3,0,0)$. Therefore, the correlation of $r_d(t)$ and $r_f(t)$ can switch sign only at a fixed level of the conditional volatility of each interest rate, which is clearly counterfactual. In contrast, in the case of $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$, equation (50) states that the conditional variance of $r_d(t)$ ($r_f(t)$) is an affine function of the square-root common factor, $Y_1(t)$, and the square-root local factor, $Y_2(t)$ ($Y_3(t)$). Thus, the two local factors, $Y_2(t)$ and $Y_3(t)$, can stochastically change the levels of the conditional volatility of $r_d(t)$ and $r_f(t)$ even though the value of $Y_1(t)$ is fixed at the intercept of the covariance function. As such, the existence of the square-root local factors can mitigate the tension of $\mathbb{IA}_{1;1,0,0}(3;3,0,0)$ in matching the sign-switching correlation and heteroskedastic volatility of the interest rates.

The advantage of $\mathbb{IQ}(3;1,1,1)$ over $\mathbb{IA}_{1;1,0,0}(3;3,0,0)$ and $\mathbb{IA}_{2;2,0,0}(3;3,0,0)$ becomes more evident by comparing the form of covariance induced by a restricted version of $\mathbb{IQ}(3;1,1,1)$ to that implied by the two affine models. Restricting the off-diagonal terms of Ψ^d, Ψ^f, and Φ_1 to zero in equations (64), (65), and (66) results in a special case of $\mathbb{IQ}(3;1,1,1)$, in which the state variables are orthogonal and there is no interaction among the state variables. From these restrictions, the covariance of $r_d(t)$ and $r_f(t)$ becomes

$$\mathrm{Cov}_{df}(t) = 4\Sigma_{11}^2\Psi_{11}^f\left(Y_1(t)+\frac{\beta_1^f}{4\Psi_{11}^f}\right)^2 - \frac{\left(\Sigma_{11}\beta_1^f\right)^2}{4\Psi_{11}^f}. \tag{73}$$

Thus the lower bound of the covariance is strictly negative. Equation (73) states that this model is more flexible than both $\mathbb{IA}_{1;1,0,0}(3;3,0,0)$ and $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$ because the correlation can switch sign when the sample path of $Y_1(t)$ passes the two intercepts: zero and -$\frac{\beta_1^f}{2\Psi_{11}^f}$. As is demonstrated in equations (69) and (70), the instantaneous variances of $r_d(t)$ and $r_f(t)$ are determined by both the common factor and the relevant local factor. Therefore, this model is free from the tension observed in $\mathbb{IA}_{1;1,0,0}(3;3,0,0)$ like $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$. It is easy to show that this restricted version of $\mathbb{IQ}(3;1,1,1)$ is more flexible than any subfamily of $\mathbb{IATSM}$s with $m_c = 1$ in inducing the sign-switching correlation of $r_d(t)$ and $r_f(t)$.

Comparing $\mathbb{IA}_{2;2,0,0}(3;3,0,0)$ and $\mathbb{IA}_{2;2,0,0}(4;2,1,1)$, with $\mathbb{IQ}(3;1,1,1)$

In both $\mathbb{IA}_{2;2,0,0}(3;3,0,0)$ and $\mathbb{IA}_{2;2,0,0}(4;2,1,1)$, the covariance of the interest rates, $r_d(t)$ and $r_f(t)$, is represented as an affine function of the two square-root common factors, $Y_1(t)$ and $Y_2(t)$. Thus to generate the sign-switching property of the correlation of $r_d(t)$ and $r_f(t)$, both models require some parametric restrictions. In $\mathbb{IA}_{2;2,0,0}(3;3,0,0)$, the signs of one or two terms of $(\delta_1^d\delta_1^f, \delta_2^d\delta_2^f, \delta_3^d\delta_3^f)$ must be negative. For the case of $\mathbb{IA}_{2;2,0,0}(4;2,1,1)$, either $\delta_1^d\delta_1^f$ or $\delta_2^d\delta_2^f$ must take negative value. Conditioning on $Y_2(t)$, these two affine models can induce the sign-switching property of the correlation when the sample path of $Y_1(t)$ passes the intercept of the covariance function: $-\frac{\delta_3^d\delta_3^f + \left(\delta_2^d\delta_2^f + \delta_3^d\delta_3^f\beta_{32}\right)Y_2(t)}{\delta_1^d\delta_1^f + \delta_3^d\delta_3^f\beta_{31}}$ for $\mathbb{IA}_{2;2,0,0}(3;3,0,0)$, and $-\frac{\delta_2^d\delta_2^f Y_2(t)}{\delta_1^d\delta_1^f}$ for $\mathbb{IA}_{2;2,0,0}(4;2,1,1)$. Thus $\mathbb{IA}_{2;2,0,0}(3;3,0,0)$ and $\mathbb{IA}_{2;2,0,0}(4;2,1,1)$ are more flexible than $\mathbb{IA}_{1;1,0,0}(3;3,0,0)$ and $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$.

The quadratic term of equation (72) states that $\mathbb{IQ}(3;1,1,1)$ can induce the sign-switching property of the correlation when the sample path of $Y_1(t)$ passes one of the two points: zero and $-\frac{1}{\Psi_{11}^f}\left[\frac{1}{2}\beta_1^f + \Psi_{11}^f\Psi_{12}^d Y_2(t) + \Psi_{13}^f Y_3(t)\right]$, at the given state vector $(Y_2(t), Y_3(t))$. In addition, $\mathbb{IQ}(3;1,1,1)$ can also generate the sign-switching correlation through the unrestricted signs of the linear and bilinear terms. Therefore, due to the existence of these multiple channels, $\mathbb{IQ}(3;1,1,1)$ is more flexible than $\mathbb{IA}_{2;2,0,0}(3;3,0,0)$ and $\mathbb{IA}_{2;2,0,0}(4;2,1,1)$ in generating the sign-switching property of the cross-country interest rate correlation.

5 Empirical Results

5.1 Data

In order to investigate the implication of the two-country term structure models for the joint dynamics of the domestic and foreign bond yields, and the foreign exchange rate, we utilize the Eurodollar and Euroyen interest rates, and the Dollar/Yen exchange rate data. The Eurocurrency interest rates are taken from the Financial Times and the exchange rate data is obtained from Morgan Stanley Capital International (MSCI), both provided by Datastream. Tuesday-to-Tuesday middle quote rates are sampled at a tri-weekly frequency over the period January 29, 1980 through November 26, 2002.

For the purpose of the analysis of the model's ability to fit the bond yields of the two countries and the exchange rate, we utilize the six-month Eurodollar and Euroyen yields, and the geometric USD/JPY return. The geometric exchange rate return is calculated from the exchange rate data sampled every three weeks. The data, which are plotted in Figure 2, offer some interesting characteristics. First, as discussed in detail later, the conditional correlation between the Eurodollar and Euroyen yields shows a pronounced sign-switching property, which is the main focus of this paper. Second, the sample period covers a wide range of interest rate regimes, from the high and volatile interest rate regime of the early 1980's in U.S. to the very low and stable rate regime of the late 1990's in Japan.[20] Third, as will be discussed, our sample includes various important economic episodes, and thus enables us to investigate their effects on the correlation of the Eurodollar and Euroyen yields.

5.2 The Efficient Method of Moments

Estimation of the international term structure models requires a sophisticated econometric methodology. First it needs to estimate the parameters of latent variables. Second, since the models are expressed in continuous time, it is necessary to avoid issues of discretization bias. Last, but not least, the estimation scheme should provide a flexible method of estimating time-varying second moments, especially correlations. The last point is, of course, particularly critical in our paper. Recent econometric advances have allowed researchers to address all of these issues through the use of the Efficient Method of Moments (EMM) that is developed by Gallant and Tauchen (1996).[21] Briefly the ideas are as follows.[22]

The EMM procedure can be thought of as a three-step procedure. The first step is to fit a consistent estimator of the conditional density of the observable data. Designate this approximation to the density as

$$\hat{f}_K\left(y_t|x_{t-1},\theta\right) = \frac{f(y_t, x_{t-1}|\theta)}{f(x_{t-1}|\theta)},$$

where x_{t-1} is the state vector of the observable process at time $t-1$, y_t denotes the current observation of the observed process, which is a vector of the Eurodollar yield, the Euroyen yield and the Dollar-Yen exchange rate return in our application, and θ denotes the $K-$dimensional parameter vector of the density approximation. Following Gallant and Tauchen (2009, 2010a, 2010b), we approximate this density using the SNP procedure. The SNP model is a vector autoregression (VAR) on L_u lags with a modified BEKK-GARCH(L_g, L_r, L_v, L_w) variance function, where the modification is the addition of leverage and level effects to the specification described in Engle and Kroner (1995). The innovation density is a Hermite density of degree K_z. A Hermite density has a form of a polynomial times the multivariate standard normal density, which product is then normalized to integrate to one. To allow for conditional heterogeneity over and above that allowed by GARCH, the coefficients of the polynomial in the Hermite density are themselves polynomials of degree K_x in L_p lags of the data. Because the number of terms in a polynomial expansion become exponentially large as the dimension increases, two additional tuning parameters are introduced to control interactions: $I_z \geq 0$ specifies the maximum degree of interactions among the y_t; similarly for I_x. The tuning parameters of an SNP model are, therefore, $\{L_u, L_g, L_r, L_v, L_w, L_p, K_z, I_z, K_x, I_x\}$. The SNP model defines a likelihood for the data. Standard maximum likelihood methods provide an estimate $\tilde{\theta}$ of its parameters and an estimate $\tilde{\mathcal{I}}$ of the information matrix. The appropriate SNP specification is determined by increasing the tuning parameters along an upward model expansion path and using the Bayes Information Criterion (BIC) due to Schwarz (1978) to select the best model along the path.

The second step in the EMM process involves estimating a parameter vector for the international

[20]The low levels of the Euroyen yield data, which are close to zero, is an outcome of the monetary policy of the Bank of Japan (BOJ). Starting from August 1995, the BOJ maintained its target rate for uncollateralized overnight call between zero and 50 basis points in an effort to facilitate economic recovery. In particular, the BOJ adopted a zero interest rate policy on February 2, 1999. This policy was maintained until March 9, 2006 except the period from August 2000 through February 2001.

[21]The EMM estimation is extended to non-Markovian data with latent variables in Gallant and Long (1997).

[22]An expository discussion of the method is in Gallant and Tauchen (2010b).

term structure model. The adequacy of a set of parameters for the term structure model is evaluated by means of the EMM criterion function, which is computed from a simulation of the term structure model as follows: Denote the parameters of the term structure model by ρ, the parameters of the SNP model estimated from the data by $\tilde{\theta}$, and the information matrix of the SNP model estimated from the data by $\tilde{\mathcal{I}}$.[23] Given ρ one simulates a long set of data from the term structure model. The score vector of the SNP model (with parameters set to $\tilde{\theta}$) is computed at each of the simulated data points. Denote the average of these scores by $m\left(\rho, \tilde{\theta}\right)$. The EMM criterion function is the quadratic form

$$m'\left(\rho, \tilde{\theta}\right) \tilde{\mathcal{I}}^{-1} m'\left(\rho, \tilde{\theta}\right).$$

Small values of the EMM criterion function are preferred. Estimates of model parameters and estimates of standard errors may be determined from an MCMC chain for ρ as described in Chernozhukov and Hong (2003). Briefly, the estimate of ρ is that value $\hat{\rho}$ from the MCMC chain for which the EMM criterion function is smallest and standard errors are proportional to the standard deviations of the MCMC chain. A test of model adequacy is provided by the test statistic

$$Tm'\left(\hat{\rho}, \tilde{\theta}\right) \tilde{\mathcal{I}}^{-1} m'\left(\hat{\rho}, \tilde{\theta}\right) \sim \chi^2_{K-J},$$

where K denotes the dimension of θ and J denotes the dimension of ρ. The EMM method is as efficient as maximum likelihood, as shown in Gallant and Long (1997). The last step in the EMM process is reprojection which we shall describe in a later section.

There are two noteworthy innovations in the EMM estimation used in the current paper over those adopted in extant term structure estimations such as Dai and Singleton (2000), Ahn, Dittmar, and Gallant (2002), Bansal and Zhou (2002), and Ahn, Dittmar, Gallant, and Gao (2003). First, we implement the EMM using the MCMC estimator proposed by Chernozhukov and Hong (2003), which has better numerical properties than EMM implemented by means of conventional derivative based hill climbing optimizers. Also, the imposition of parametric restrictions and support conditions is far easier with the MCMC approach. We use a random walk, single move, normal proposal density. If a group of parameters in the MCMC chain is highly correlated then a group move scheme is used by moving that group as a whole with a multivariate normal proposal density.[24] The second innovation is the use of a BEKK-GARCH variance function for the SNP density rather than an R-GARCH variance function. The R-GARCH has a drawback of losing information on the sign of residuals since they enter the R-GARCH through their absolute values. BEKK-GARCH is particularly attractive here because estimating conditional correlations among yields is the focal point of our paper.

5.3 Choice of the SNP model

As discussed above, we select the tuning parameters of the SNP model by moving along an upward expansion path using the Schwartz (1978) BIC criterion to guide the search. The SNP densities with small BIC values are preferred. Our Schwartz preferred SNP fit is described by $\{L_u, L_g, L_r, L_v, L_w, L_p, K_z, I_z, K_x, I_x\} = \{1, 1d, 1d, 0, 1d, 1, 4, 0, 0, 0\}$. $L_u = 1$ implies that one lag of the data is sufficient to describe the mean dynamics in the VAR. $L_g = 1d$, $L_r = 1d$, $L_v = 0$, and $L_w = 1d$ imply that a diagonal BEKK-GARCH specification augmented with level effect

[23] These are maximum likelihood estimates.

[24] This strategy is often called Metropolis within Gibbs.

describes the innovations to the process. $L_p = 1$ is the minimum required by the SNP C++ implementation but its value is irrelevant here because we set $K_x = 0$ below. $K_z = 4$ suggests that a fourth-order Hermite polynomials captures deviations from normality. $K_x = 0$ suggests that it is unnecessary to incorporate lags of the process in modeling the coefficients of the Hermite polynomial. Finally, $I_z = 0$ and $I_x = 0$ imply that interaction terms are suppressed.

In the following two subsections we report and discuss various diagnostics for the $\mathbb{IATSM}$s and $\mathbb{IQTSM}$ considered in this paper. In particular, we focus on tests of goodness of fit using the EMM procedure developed by Gallant and Tauchen (1996) and qualitative analysis exploiting the reprojection analysis provided by Gallant and Tauchen (1998).

Before presenting estimation results, we discuss an issue of the identification of the market prices of risks. In the estimation of both $\mathbb{IATSM}$s and $\mathbb{IQTSM}$, we impose an empirical identification condition on the market prices of risks. As noted by Ahn, Dittmar, Gallant, and Gao (2003) and Dai and Singleton (2003), the market prices of risks explain the cross-sectional dispersion in the prices of the bonds with different maturities. Because we estimate parameters by exploiting only single bond for each country, we cannot identify the market prices of risks. To handle this problem, we fix the market prices of foreign local factor risks to zero.

5.4 EMM specification tests

(1) $\mathbb{IATSM}$s

The EMM estimation results are summarized in Table 2. The first four columns of Table 2 present the parameter estimates and the goodness of fit tests for the $\mathbb{IATSM}$s considered in this paper, $\mathbb{IA}_{1;1,0,0}(3;3,0,0)$, $\mathbb{IA}_{2;2,0,0}(3;3,0,0)$, $\mathbb{IA}_{2;2,0,0}(4;2,1,1)$, and $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$. The bottom rows of Table 2 present χ^2 statistics for model fit and a z-statistic for model fit that is asymptotically standard normal and adjusted for degrees of freedom.[25]

The z-statistics of the four $\mathbb{IATSM}$s suggest that all models are rejected by the data, which indicates that the $\mathbb{IATSM}$s estimated in this paper are incapable of capturing the joint dynamics of the Eurodollar yield, the Euroyen yield, and the USD/JPY exchange rate return. Even though all four models are rejected, $\mathbb{IA}_{2;2,0,0}(4;2,1,1)$ shows the best performance followed by $\mathbb{IA}_{2;2,0,0}(3;3,0,0)$, $\mathbb{IA}_{1;1,0,0}(3;3,0,0)$, and $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$. This ranking of the overall performance among the models leads to some interesting implications.

First, the worst performance of $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$ clearly indicates that allowing the negative correlations among the state variables is important in matching the characteristics of the data captured by the fitted SNP density. As discussed previously, $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$ is the most restrictive model in accommodating the negative correlations among the state variables.

Second, Table 2 indicates that both $\mathbb{IA}_{2;2,0,0}(3;3,0,0)$ and $\mathbb{IA}_{2;2,0,0}(4;2,1,1)$ perform better than $\mathbb{IA}_{1;1,0,0}(3;3,0,0)$. This result shows the importance of the flexibility in generating time-varying volatility of bond yields. In the case of $\mathbb{IA}_{1;1,0,0}(3;3,0,0)$, only one square-root common factor, $Y_1(t)$, can contribute to inducing the heteroskedastic volatility of the two bond yields, while two factors, $Y_1(t)$, and $Y_2(t)$, derive the volatility of the yields in both $\mathbb{IA}_{2;2,0,0}(3;3,0,0)$ and $\mathbb{IA}_{2;2,0,0}(4;2,1,1)$.

[25]The z-statistic is calculated as $\frac{\chi^2 - df}{\sqrt{2df}}$ and represents a degrees of freedom normalization of the χ^2 statistic.

Third, although $\mathbb{IA}_{2;2,0,0}(3;3,0,0)$ improves upon the specifications of both $\mathbb{IA}_{1;1,0,0}(3;3,0,0)$ and $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$, it slightly underperforms $\mathbb{IA}_{2;2,0,0}(4;2,1,1)$. The loss of degrees of freedom of $\mathbb{IA}_{2;2,0,0}(4;2,1,1)$ is compensated by the improvement in fit. The main difference between $\mathbb{IA}_{2;2,0,0}(3;3,0,0)$ and $\mathbb{IA}_{2;2,0,0}(4;2,1,1)$ is that $\mathbb{IA}_{2;2,0,0}(4;2,1,1)$ can accommodate the country-specific movements of the data, which cannot be captured by $\mathbb{IA}_{2;2,0,0}(3;3,0,0)$. Other characteristics of these two models are similar. In both models, two common square-root factors derive the heteroskedatic volatility of the state vector. The factor structure of $\mathbb{IA}_{2;2,0,0}(4;2,1,1)$ is characterized by the two square-root factors, $Y_1(t)$ and $Y_2(t)$, which are common to both local markets, and the two Gaussian factors, $Y_3(t)$ and $Y_4(t)$, where $Y_3(t)$ is the domestic local factor and $Y_4(t)$ is the foreign local factor, respectively. As such, $\mathbb{IA}_{2;2,0,0}(4;2,1,1)$ is similar to $\mathbb{IA}_{2;2,0,0}(3;3,0,0)$ in the flexibility of generating the negative correlations among the state variables. For $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$, it is of interest to investigate the estimates of the sensitivities, δ_3^d and δ_4^f, of the two short rates to the domestic and foreign local factors. As shown in the third column of Table 2, the sign of the estimated sensitivity, δ_3^d, of the Eurodollar short rate, $r_d(t)$, to the domestic local factor is positive and significant. In contrast, the estimated value of the sensitivity, δ_4^f, of the Euroyen short rate, $r_f(t)$, to the foreign local factor is close to zero and not significant. Thus, the foreign local factor of $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$ mainly impacts the dynamics of the USD/JPY return rather than that of the Euroyen yield.

To get additional insight into the performance of the models, we analyze the t-ratios for the scores of the best model fit with respect to the SNP parameters. The first four columns of Table 3 present the t-ratios for the 39 moment conditions for the $\mathbb{IATSM}$s. With these diagnostic t-ratios, we can analyze the strengths and weaknesses of the different model specifications. Different elements of the score correspond to different characteristics of the data. If a given two-country term structure model is capable of matching a particular score, then the t-ratio for that score should not be large. As suggested by Gallant and Long (1997) and Tauchen (1998), a t-ratio above 2.0 in magnitude indicates that the model fails to fit the corresponding score.

In Table 3, there are three sets of parameters. The parameters $b_0(1)$-$B(3,3)$ represent the VAR terms in the SNP auxiliary model, and thus represent the conditional mean of the data. The parameters $R_0(1,1)$-$W_1(3,3)$ are the modified BEKK-GARCH terms, which model the conditional volatility of the data. The $a_0(1)$-$a_0(12)$ terms are the Hermite polynomial parameters, which represent the shape characteristic of the density for the data. Specifically, the parameters $a_0(1)$-$a_0(4)$ denote the Hermite polynomial terms for the USD/JPY return, $a_0(5)$-$a_0(8)$ denote the Hermite polynomial terms for the Euroyen yield, and $a_0(9)$-$a_0(12)$ denote the Hermite polynomial terms for the Eurodollar yield.

All of the affine models estimated in this paper perform quite well in capturing the mean dynamics of the VAR part of the SNP density. In particular, none of the t-ratios with respect to the VAR terms for the USD/JPY return as well as for the Eurodollar and Euroyen yields are larger than 2.0 in magnitude. However, it should be noted that our result does not exclude the possibility for the existence of the non-bond factors affecting the dynamics of the foreign exchange rate since we estimate the models by simultaneously utilizing the bond yield data and the exchange rate return data.[26]

As presented in Table 3, $\mathbb{IA}_{1;1,0,0}(3;3,0,0)$ has more difficulty than other affine models in describ-

[26]Brandt and Santa-Clara (2002), Inci and Lu (2004), Mosburger and Schneider (2005), and Leippold and Wu (2007) incorporate non-bond factors to model the dynamics of the foreign exchange rate.

ing the GARCH part of the SNP model. Of the fifteen GARCH scores, four are significant for $\mathbb{IA}_{1;1,0,0}(3;3,0,0)$. The remaining three affine models, $\mathbb{IA}_{2;2,0,0}(3;3,0,0)$, $\mathbb{IA}_{2;2,0,0}(4;2,1,1)$, and $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$, have similar performances in fitting the GARCH effect. There are three scores with respect to the GARCH terms that are larger than 2.0 in absolute value for each of these three affine models. This result indicates that, although $\mathbb{IA}_{2;2,0,0}(3;3,0,0)$, $\mathbb{IA}_{2;2,0,0}(4;2,1,1)$, and $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$ perform better than $\mathbb{IA}_{1;1,0,0}(3;3,0,0)$, they have still some difficulties in matching the conditional volatility of the data. However, all of the affine models are able to fit the level effect of the volatility dynamics. None of the t-ratios with respect to $W_1(1,1)$, $W_1(2,2)$, and $W_1(3,3)$ are significant for all four $\mathbb{IATSM}$s.

Investigating the t-ratios for the scores with respect to the Hermite terms $a_0(5)$-$a_0(12)$ reveals that $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$ is the worst-performing model in fitting the shape of the density for the two bond yields. $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$ produces three significant Hermite term scores. All of the remaining three $\mathbb{IATSM}$s, $\mathbb{IA}_{1;1,0,0}(3;3,0,0)$, $\mathbb{IA}_{2;2,0,0}(3;3,0,0)$, and $\mathbb{IA}_{2;2,0,0}(4;2,1,1)$, have t-ratios greater than 2.0 only for $a_0(10)$. This indicates that all of the models $\mathbb{IA}_{1;1,0,0}(3;3,0,0)$, $\mathbb{IA}_{2;2,0,0}(3;3,0,0)$, and $\mathbb{IA}_{2;2,0,0}(4;2,1,1)$ fail to fit the shape characteristics of the density for the Eurodollar yield, while they perform fairly well in capturing the deviations from conditional normality of the Euroyen yield. The worst performance of $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$ indicates that the flexibility in generating the negative correlations among the state variables is quite important for capturing the shape characteristics of the conditional density for the Eurodollar and Euroyen bond yield data.[27] In $\mathbb{IA}_{1;1,0,0}(3;3,0,0)$, the estimate of κ_{31} is 4.2783 and statistically significant, which means that the first Gaussian common factor, $Y_1(t)$, and the square-root common factor, $Y_3(t)$, are negatively correlated. Similarly, the estimate of κ_{32} for $\mathbb{IA}_{2;2,0,0}(3;3,0,0)$ is 1.6760 and significant. Thus, the second square-root common factor, $Y_2(t)$, and the Gaussian common factor, $Y_3(t)$, are negatively correlated in $\mathbb{IA}_{2;2,0,0}(3;3,0,0)$. In case of $\mathbb{IA}_{2;2,0,0}(4;2,1,1)$, the signs of the estimates of κ_{32} and κ_{42} are significantly positive. This indicates that the second square-root common factor, $Y_2(t)$, is negatively correlated with both the Gaussian domestic local factor, $Y_3(t)$, and the Gaussian foreign local factor, $Y_4(t)$, in $\mathbb{IA}_{2;2,0,0}(4;2,1,1)$. In contrast, our parameter estimates reveal that $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$ is unable to accommodate the negative correlations among the state variables because the sign of the estimate of κ_{41}, the only channel through which $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$ can allow the negative correlations among the four state variables, is significantly negative. Finally, the t-ratios for the scores with respect to $a_0(1)$-$a_0(4)$ indicate that $\mathbb{IA}_{1;1,0,0}(3;3,0,0)$, $\mathbb{IA}_{2;2,0,0}(4;2,1,1)$, and $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$ performs better than $\mathbb{IA}_{2;2,0,0}(3;3,0,0)$ in matching the shape characteristics of the USD/JPY return.

(2) $\mathbb{IQTSM}$

The final column of Table 2 presents estimates for $\mathbb{IQ}(3;1,1,1)$. As indicated by the z-statistic, $\mathbb{IQ}(3;1,1,1)$ provides a much better fit to the data than the four $\mathbb{IATSM}$s. The z-statistic of $\mathbb{IQ}(3;1,1,1)$ is 1.852 whereas the z-statistic of $\mathbb{IA}_{2;2,0,0}(4;2,1,1)$, which is the best performing $\mathbb{IATSM}$, is 4.573.

The last column of Table 3 presents the diagnostic t-ratios for $\mathbb{IQ}(3;1,1,1)$. It is surprising that there is only one t-ratio exceeding 2.0 in magnitude. Thus, the diagnostics suggest that

[27] In their single country affine setting, Dai and Singleton (2000) find that accommodating the negative correlations among the state variables is important in matching the higher moments of U.S. bond yields. Ahn, Dittmar, and Gallant (2000) and Ahn, Dittmar, Gallant, and Gao (2003) demonstrate that this role of negative correlations among the state variables is also valid for their single country non-affine models.

$\mathbb{IQ}(3;1,1,1)$ is capable of capturing most of the features of our trivariate data implied by the SNP model. First, like the four $\mathbb{IATSM}$s, $\mathbb{IQ}(3;1,1,1)$ is able to fit the VAR part of the SNP model. Second, unlike the affine models, $\mathbb{IQ}(3;1,1,1)$ performs fairly well in matching the scores with respect to the GARCH terms. As discussed above, all of the $\mathbb{IATSM}$s have some difficulties in matching the GARCH scores. Third, $\mathbb{IQ}(3;1,1,1)$ performs better than the four $\mathbb{IATSM}$s in capturing the shape characteristic of the density for the Eurodollar and Euroyen yields. None of the t-ratios for the scores with respect to the Hermite terms for the two bond yields, $a_0(5)$-$a_0(12)$, exceed 2.0. Among the affine models, $\mathbb{IA}_{1;1,0,0}(3;3,0,0)$, $\mathbb{IA}_{2;2,0,0}(3;3,0,0)$, and $\mathbb{IA}_{2;2,0,0}(4;2,1,1)$ fail to fit the higher moments of the Eurodollar yield, and $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$ is able to fit none of the Hermite terms of the two bond yields. Fourth, however, $\mathbb{IQ}(3;1,1,1)$ cannot adequately describe the shape characteristics of the density for the USD/JPY return. Of the four scores with respect to the Hermite terms, the t-ratio for $a_0(3)$ exceeds 2.0 in absolute value. Thus, none of the two-country models estimated in this paper are capable of matching the deviations from conditional normality of the USD/JPY return.

We conclude this section by investigating the lower bounds for the U.S. and Japanese interest rates implied by the parameter estimates for $\mathbb{IQ}(3;1,1,1)$. As discussed, among the five models considered in this paper, only $\mathbb{IQ}(3;1,1,1)$ can guarantee the positivity of the two short rates, $r_d(t)$ and $r_f(t)$. Our parameter estimates lead to the lower bound for the Eurodollar interest rate, $r_d(t)$, of 1.51 percent, which is greater than the minimum of the observed six-month Eurodollar yield, 1.42 percent. The lower bound for the Euroyen interest rate, $r_f(t)$, implied by $\mathbb{IQ}(3;1,1,1)$ is 0.01 percent, which is smaller than the observed minimum of the six-month Euroyen yield, 0.05 percent. This result indicates that although $\mathbb{IQ}(3;1,1,1)$ overestimates the lower bound for the Eurodollar interest rate, it can generate a reasonable lower bound for the Euroyen interest rate, which is close to the zero bound.

5.5 Reprojection

The reprojection method of Gallant and Tauchen (1988) provides additional diagnostics for the adequacy of the $\mathbb{IATSM}$s and $\mathbb{IQTSM}$. A detailed discussion of the method is provided in their paper. The idea behind the method is to compare the conditional density for discretely sampled data that is implied by the structural model to a conditional density computed directly from the data. As discussed, however, closed-form solutions are not available for the conditional density implied by the international term structure models considered in this paper. Gallant and Tauchen (1988) propose to generate a large simulation from the structural model wherein the structural parameters are set to the EMM estimates, and to fit an SNP model to the simulated data. Gallant and Long (1997) prove, under regularity conditions, that the SNP density thus estimated converges to the conditional density implied by the structural model.

Of immediate interest in eliciting the dynamics of observables are the first two one-step-ahead conditional moments

$$\mathcal{E}(y_0|y_{-L},\ldots,y_{-1}) = \int y_0\, f_K(y_0|x_{-1},\hat{\theta}_K)\, dy_0$$

and

$$\begin{aligned}&\mathrm{Var}(y_0|y_{-L},\ldots,y_{-1})\\ &= \int [y_0 - \mathcal{E}(y_0|x_{-1})]\,[y_0 - \mathcal{E}(y_0|x_{-1})]'\; f_K(y_0|x_{-1},\hat{\theta}_K)\, dy_0\end{aligned}$$

where $x_{-1} = (y_{-L}, \ldots, y_{-1})$.

Figures 3-17 compare the first two conditional moments, excluding the conditional correlation between the Eurodollar and Euroyen yields, implied by the SNP model fitted to the data to those implied by $\mathbb{A}_{1;1,0,0}(3;3,0,0)$, $\mathbb{A}_{2;2,0,0}(3;3,0,0)$, $\mathbb{A}_{2;2,0,0}(4;2,1,1)$, $\mathbb{A}_{3;1,1,1}(4;2,1,1)$, and $\mathbb{Q}(3;1,1,1)$. The figures depict the conditional moments implied by the international term structure models (dashed lines) and the conditional moments implied by the SNP model (solid lines). Reprojection results for the conditional correlation between the two yields are presented in Section 7.

Plots of the conditional mean implied by the five international term structure models are presented in Figures 3 through 7. All of the models do quite well in reproducing the mean dynamics of the two yields, and are successful in matching the extremely low level of the Euroyen yield between the late 1990s and 2001. Especially, $\mathbb{A}_{2;2,0,0}(3;3,0,0)$, $\mathbb{A}_{2;2,0,0}(4;2,1,1)$ and $\mathbb{A}_{3;1,1,1}(4;2,1,1)$ can almost completely duplicate the conditional mean of the two bond yields. However, the models show differences in matching the conditional mean of the USD/JPY return. The conditional mean of the USD/JPY return implied by $\mathbb{A}_{2;2,0,0}(3;3,0,0)$ and $\mathbb{A}_{3;1,1,1}(4;2,1,1)$ is smoother than the projected conditional mean. However, $\mathbb{A}_{1;1,0,0}(3;3,0,0)$, $\mathbb{A}_{2;2,0,0}(4;2,1,1)$, and $\mathbb{Q}(3;1,1,1)$ provide a reasonably accurate description of the conditional first moment of the USD/JPY return.

Figures 8-12 compare the conditional volatility of the data to that implied by the data. First, among the four affine models, $\mathbb{A}_{1;1,0,0}(3;3,0,0)$ performs worst in tracking the volatility of the data. The conditional volatility of the two yields reproduced by $\mathbb{A}_{1;1,0,0}(3;3,0,0)$ is almost flat while the projected volatility show pronounced time-varying property. Interestingly, $\mathbb{A}_{1;1,0,0}(3;3,0,0)$ is able to reproduce the shape of the USD/JPY return volatility. However, it fails to match the level of the volatility. Second, $\mathbb{A}_{2;2,0,0}(3;3,0,0)$, $\mathbb{A}_{2;2,0,0}(4;2,1,1)$, and $\mathbb{A}_{3;1,1,1}(4;2,1,1)$ are similar in reproducing the conditional volatility of the two yields. They are able to track the volatility of the Eurodollar yield including the high volatility in the early 1980s. However, they have some difficulty in tracking the volatility of the Euroyen yield. Although both $\mathbb{A}_{2;2,0,0}(3;3,0,0)$ and $\mathbb{A}_{3;1,1,1}(4;2,1,1)$ are able to fit the shape of the USD/JPY return volatility, they underestimate the level of the USD/JPY return volatility. On the contrary, $\mathbb{A}_{2;2,0,0}(4;2,1,1)$ is quite well in tracking the volatility of the USD/JPY return. Third, $\mathbb{Q}(3;1,1,1)$ shows a surprising performance. It can almost perfectly track the volatility of the Eurodollar yield and the USD/JPY return. In addition, $\mathbb{Q}(3;1,1,1)$ performs better than the four affine models in matching the volatility of the Euroyen yield. However, it overestimates the level of the volatility in the mid-1980s and in the early 1990s.

Figures 13-18 depict the conditional correlation among the USD/JPY return and the two bond yields, which is crucial in the management of the currency-unhedged international bond portfolios. The results are promising. Excepting $\mathbb{A}_{1;1,0,0}(3;3,0,0)$, the affine models are able to capture the correlation dynamics. The two conditional correlations implied by $\mathbb{A}_{1;1,0,0}(3;3,0,0)$ are much smoother than those implied by the fitted SNP model. Although the remaining three affine models, $\mathbb{A}_{2;2,0,0}(3;3,0,0)$, $\mathbb{A}_{2;2,0,0}(4;2,1,1)$, and $\mathbb{A}_{3;1,1,1}(4;2,1,1)$, are quite well, the performance of $\mathbb{Q}(3;1,1,1)$ is better. $\mathbb{Q}(3;1,1,1)$ is able to exactly track the two correlations excepting a slight overestimate of the correlation between the Euroyen yield and the USD/JPY return in the 1980s.

In summary, the results of the reprojection analysis conform largely to the results of the EMM specification tests. $\mathbb{IQ}(3;1,1,1)$ performs better than the four affine models in capturing the conditional second moments of the data. Although, $\mathbb{IQ}(3;1,1,1)$ is able to reproduce the mean dynamics of the data, three affine models, $\mathbb{IA}_{2;2,0,0}(3;3,0,0)$, $\mathbb{IA}_{2;2,0,0}(4;2,1,1)$, and $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$, perform better than $\mathbb{IQ}(3;1,1,1)$ in matching the conditional mean of the Euroyen yield.

6 Diagnostics on the quality of SNP estimates of correlations

In this section, we present a qualitative analysis of the characteristics of the conditional correlation between the six-month Eurodollar and Euroyen yields, which is implied by the preferred SNP density, $\{1, 1d, 1d, 0, 1d, 1, 4, 0, 0, 0\}$. Because the conditional correlations are not observable, the SNP estimates of them *per se* are not free from a potential misspecification error of the SNP model. This problem is particularly critical in our paper since the SNP estimates are used as an (auxiliary) 'unrestricted' benchmark for the estimates implied by the theoretical models, $\mathbb{IATSM}$s and $\mathbb{IQTSM}$s. Thus it is desirable to diagnose the admissibility of the SNP estimates themselves, i.e., whether their time varying behavior is economically meaningful. Unfortunately there is no formal way of doing that due to the unobservability of conditional correlations. As an alternative, we diagnose them by investigating whether the extreme values of the SNP estimates of correlations are supported by *historical events*. This heuristic analysis is particularly suggestive since the data period includes the eras marked by a score of important historical events between the U.S. and Japan. Thus it enables us to better understand the quality of the SNP estimates of correlations and identify major economic driving forces behind the co-movement of the Eurodollar and Euroyen yields.

Figure 18 shows the projected time-varying correlation between the Eurodollar and Euroyen yields from the SNP model. Projected correlation is the one-step ahead conditional correlation of the SNP projection.[28] The correlation fluctuates widely between -56 percent and 79 percent and shows dramatic changes over time. While the two yields were positively correlated for most of time until the mid-1990s, there are three noticeable drops in the correlation in the second half of the 1980s. During those three episodes, the two bond yields become negatively correlated. For the period between the second half of the 1990s and 2002, the correlation shows pronounced sign-switching oscillation on the back of losing directionality. Interestingly, there is a sharp rise in the correlation around 2001 resulting in a strong coupling in the movements of the two bond yields.

To investigate the link between the correlation and the economic fundamentals in detail, we classify the correlation into the three regimes: high positive correlation, negative correlation, and sign-switching oscillation regimes. Their respective operational definitions are as follows:

1. (Regime-P) Strong positive correlation
   Any period during which the SNP correlation is larger than 50 percent at least for one month. Because our sampling frequency is every three weeks, this requirement means that the correlation should be larger than 50 percent in at least three consecutive sample points.

   There are six periods for the high positive correlation regime: from December 1981 to

[28]Because we use 28 initial lags for the SNP estimation, our analysis in this section covers the period from September 8, 1981 to November 26, 2002.

January 1982 (P1), from November 1982 to March 1983 (P2), from June 1983 to September 1983 (P3), from November 1987 to May 1988 (P6), from August 1988 to October 1988 (P7), and from February 2001 to July 2001 (period 10)

2. (Regime-N) Negative correlation
   Any period during which the SNP correlation estimate is below -5 percent but do not revisit the negative territory in the following three months after returning to the positive value.

   There are three periods: October 1985 (P4), from April 1987 to September 1987 (P5), and from June 1989 to November 1989 (P8).

3. (Regime-O) Sign-switching oscillation
   Any period during which the SNP correlation estimate hits negative value and revisit the negative territory in the following three months after returning to the positive value

   There are two periods: from November 1993 to November 2000 (P9) and from January 2002 to November 2002 (P11).

In the following subsections, we present a qualitative investigation of the link between the SNP correlation and economic fundamentals of each country. To do so, we re-group the above-mentioned 11 periods by three potential determinants of time-varying correlations: business cycles, (non-business cycle driven) monetary policies and market's expectation about future monetary policies. We focus exclusively on monetary policy related drivers since the Eurodollar and the Euroyen yields are short-end rates, which are most susceptible to a change in monetary policies of the U.S. and Japan.

- Synchronization and dis-synchronization of business cycles
  Synchronization and dis-synchronization of business cycles may result in coordination or discoordination of monetary policies between the U.S. and Japan. We directly investigate business cycles, which are more fundamental.

- Coordination and dis-coordination of monetary policies
  This driver refers to coordination and dis-coordination of monetary policies which are <u>not</u> related to business cycles. Such a move in monetary policies may be driven by international accords or financial market crashes which do not result in economic recession.

- Market's expectation about future monetary policies
  Market participants may price in their expectation about future monetary policies.

6.1 Synchronization and dis-synchronization of business cycles: P1, P2, P9, P10 and P11

Erb, Harvey, and Viskanta (1994) and Longin and Solnik (1995) state that the cross-country equity correlations of major industrialized countries tend to be related to the macroeconomic fluctuations in the respective countries. In particular, the empirical analysis of Erb, Harvey, and Viskanta (1994) shows that the correlations of the G7 countries' equity returns are related to the coherence between business cycles. They report that the cross-country equity correlations are higher than usual when two countries are in recession.

How about the cross-country correlation of bond yields? In a single country setting, many studies have reported that the dynamics of the U.S. bond prices are related to the business cycle.[29] However, little is known about the relationship between the international business cycle linkages and the cross-country correlations of bond yields. In the following analysis, we present a qualitative evidence that the correlation of the Eurodollar and Euroyen yields reflects the synchronization of the business cycles of U.S. and Japan.

(1) Synchronized recessions: P1, P2, and P10

Figure 19 presents the business cycles of the U.S. and Japan. The upper shaded area represents recessions in the U.S. defined by the NBER (National Bureau of Economic Research) while the rest periods are defined as boom. According to the NBER reference business cycle, there were four recessions in the U.S. since 1980: from January 1980 to July 1980, from July 1981 to November 1982, from July 1990 to March 1991, and from March 2001 to November 2001.[30]

Similarly, the bottom shaded area indicates recession in Japan defined by the ESRI (Economic and Social Research Institute of the Cabinet Office of the Japanese government). According to the ESRI's official business cycle dating, there were five recessions in Japan since 1980: from February 1980 to February 1983, from June 1985 to November 1986, from February 1991 to October 1993, from May 1997 to January 1999, and from November 2000 to January 2002. Similar to the NBER's business cycle dating for U.S., the ESRI's business cycle dating is considered as the official reference dates for the Japanese business cycle [see, e.g. Okina, Shirakawa, and Shiratsuka (2001), Hayashi and Prescott (2002), and Wall (2006)].[31]

Figure 19 indicates that there are three recession shared by the two countries: the early 1980s, the early 1990s and 2001. As a matter of fact, those three synchronized recessions are counted as 'global' recessions shared by major developed countries.[32] Figure 19 reveals that the two yields were closely coupled during the recessions in the early 1980s and in 2001, but less so in the early 1990s. First, between September 1981 and November 1982, in which both countries were in recession, the correlation between the two yields showed fluctuation between 19 percent and 74 percent. Despite the fluctuation, however, the Eurodollar and Euroyen yields had positive correlation greater than 45 percent in most of time except the period March 1982 through June 1982. Interestingly P1 and P2 belong to this period. Second, the two bond yields were strongly coupled between March 2001 and November 2001 when the U.S. and Japanese economy entered into recession on the back of the IT burble burst. Especially, the correlation was larger than 50 percent during the consecutive 6 months from March 2001 to August 2001. This period nests P10. Third, compared to the recessions in the early 1980s and in 2001, the recessions in the early 1990s shared by all G7 countries led to much weaker positive correlation

[29] See, e.g. Fama (1986), Stambaugh (1988), Fama and French (1989), Ang and Bekaert (2002), Bansal and Zhou (2002), Duffee (2009), Campbell and Diebold (2009), and Ludvigson and Ng (2009). Recently, a growing number of papers incorporate macro factors in their modeling of the U.S. term structure of interest rates, and find that macroeconomic fluctuations are important in explaining term premia [See, e.g. Ang and Piazzesi (2003), Rudebusch and Wu (2008), Joslin, Priebsch, and Singleton (2009), and Ang, Bolvin, Dong, and Loo-Kung (2009)].

[30] The recession from January 1980 to July 1980 is not plotted in Figure 19. The reference dates of the NBER business cycle can be found at www.nber.org/cycles.html

[31] Many researches use the ESRI business cycle as the benchmark in their modeling of the business cycle of Japan [See, e.g. Watanabe (2003) and the references therein]. Details of the ESRI's dating method and the reference dates of business cycle can be found at www.esri.cao.go.jp/en/stat/di/di2e.html.

[32] See, among many others, Gregory, Head, Raynauld (1997), Helbling and Bayoumi (2003,) Kose, Otrok, and Whiteman (2008), Canova, Ciccarelli, and Ortega (2007), and Claessens, Kose, and Terrones (2008)

between the Eurodollar and Euroyen yields. In summary, out of the three synchronized recessions, the first and the third result in strong positive correlations in yields (P1, P2 and P10). The second synchronization was short-lived as shown in Figure 19, and did not induce strong positive correlations.

It is an interesting question to see whether Figure 19 can be reconciled with the findings of the existing literature on international business cycles.[33] Using dynamic factor models, Helbling and Bayoumi (2003), Canova, Ciccarelli, and Ortega (2007), and Kose, Otrok, and Whiteman (2008) find that among the three main recessions in our sample period, the recessions in the early 1980s and in 2001 were closely synchronized across the G-7 countries in the sense that the fluctuations of various macroeconomic aggregates showed strong correlations with common shocks. In contrast, their results suggest that the recessions in the early 1990s were the least synchronized, especially for the case of Japan. For instance, Kose, Otrok, and Whiteman (2008) employ a Bayesian dynamic factor model and evaluate the roles of G-7 common and country-specific factors in capturing the changes in G-7 business cycles. Their results suggest that the U.S. and Japan country factors, and the G-7 factors showed strong co-movments, and all factors were indicating economic contradictions during the recessions in the early 1980s and in 2001. However, between 1990:3Q and 1991:1Q when the U.S. economy was in recession and the Japanese economy was expanding, the Japan country factor was indicating an economic boom, while the U.S. and G-7 factors were signaling a contradiction.

In contrast, between 1991:1Q and 1993:4Q when the Japanese economy was in recession and the U.S. economy bottomed out, the Japan and G-7 factors moved closely and both factors were indicating an economic downturn, while the U.S. country factor was booming.[34] Thus, our result that the Eurodollar and Euroyen yields were more strongly coupled during the two recessions in the early of 1980s and in 2001 than the recessions in the early 1990s seems to be largely consistent with their findings.

In summary, Figure 19 suggests that the dynamics of the cross-country correlation of the bond prices is strongly associated with the co-movements of the cyclical macroeconomic fluctuations of U.S. and Japan. The Eurodollar and Euroyen yields showed high positive correlations during the two recessions in the early 1980s and in 2001 reflecting the highly synchronous nature of them. Interestingly, reflecting the low synchronicity of the recessions in the early 1990s, the two yields showed weaker positive correlations.

(2) Is coupling in bond markets stronger during recessions?

As shown in Figure 19 and suggested above, correlations between the Eurodollar and Euroyen yields are not conspicuously high when both countries are in growth cycles than when they are

[33]A central issue in this field is the evolution of the degree of business cycle co-movements among major advanced countries. The existing studies report mixed results depending on their samples and econometric methods. Helbling and Bayoumi (2003), Heathcote and Perri (2004), and Stock and Watson (2005) find that the correlations of the G-7 business cycles have decreased from the late 1980s. On the contrary, Kose, Otrok, and Whiteman (2008) find that the degree of business cycle synchronization across the G-7 countries has increased during the period 1986 to 2003 than the periods before 1986. Similarly, Canova, Ciccarelli, and Ortega (2007) find that the strength of synchronization across the G-7 countries has increased from 1980s to 1990s.

[34]See Figures 1a, 1b, and 1d of Kose, Otrok, and Whiteman (2005), which is an earlier version of Kose, Otrok, and Whiteman (2008). See also Figure 2 of Canova, Ciccarelli, and Ortega (2007) for similar results. Helbling and Bayoumi (2003) demonstrates that the business cycles of the G-7 countries in the episode of the early 1990s recessions showed dis-synchronization due to large country-specific shocks.

simultaneously experiencing recession.[35] If so, the result is similar to the finding of Erb, Harvey and Viskanta (1994) which documents that the correlations of the developed countries' equity returns are higher during recession than during expansion.

To investigate this hypothesis formally, we compute average correlation for a different phase of joint business cycles. Specifically, based on the NBER and ESRI business cycles, we compare the average of the SNP correlation in four different regimes of the two-country business cycles: recession in U.S.-recession in Japan (RR), expansion in U.S.-expansion in Japan (EE), expansion in U.S.-recession in Japan (ER) and finally recession in U.S.-expansion in Japan (RE). We conduct the following simple regression:

$$\hat{\rho}_t = \alpha + \beta_{EE} I_{EE,t} + \beta_{RE} I_{RE,t} + \beta_{ER} I_{ER,t} + \epsilon_t, \tag{74}$$

where

$$\begin{aligned} I_{EE,t} &= \begin{cases} 1 & \text{if the U.S. in expansion and Japan in expansion} \\ 0 & \text{otherwise} \end{cases} \\ I_{RE,t} &= \begin{cases} 1 & \text{if the U.S. in recession and Japan in expansion} \\ 0 & \text{otherwise} \end{cases} \\ I_{ER,t} &= \begin{cases} 1 & \text{if the U.S. in expansion and Japan in recession} \\ 0 & \text{otherwise} \end{cases} . \end{aligned}$$

Because $\hat{\rho}_t$ is the SNP estimate, the regression coefficients are inefficient, but not biased. However, its feasible value is $[-1, 1]$ so that we will adopt the Generalized Method of Moments (GMM) estimation with serial correlation and heteroskedasticity-adjustment using Newey and West (1987). The estimation results are tabulated in Table 4. α, average correlation when both countries are in recession (RR), is about 0.45, which is strongly positive. In contrast, average correlations in EE, RE and ER regimes are 0.1306 ($\alpha + \beta_{EE}$), 0.3530 ($\alpha + \beta_{RE}$) and 0.1995 ($\alpha + \beta_{ER}$) respectively. All of β coefficients, which measure difference in average correlations from RR, are statistically significantly negative; so the two countries' yields are most strongly coupled during synchronized recession eras, which is in line with the similar finding of Erb, Harvey and Viskanta (1994) in equity space.

Table 4 also shows the results of Wald tests on two hypotheses: $\beta_{RE} = \beta_{ER}$ and $\beta_{ER} - \beta_{EE} = 0$. We find that β_{RE} is greater than β_{ER} and also β_{ER} is greater than β_{EE} with statistical significance. Consequently, we can conclude that the degree of coupling in yields weakens in order of RR, RE, ER and EE, which means asymmetric correlations across the different phases of international business cycles. Specifically, the two short-end bond markets are more likely to be coupled when the U.S. is in recession and Japan is in expansion than when the U.S. is in expansion and Japan is in recession. That is, when the U.S. makes a rate cut on the back of recession, Japan is more likely to follow such a move even when Japan is not in explicit recession; However the opposite is less likely so the Fed is a leader and the BOJ is a follower during monetary easing. The yields are least coupled when both economies are in expansion cycle. So during a global expansion period, each monetary authority may have a more room for conducting an independent monetary policy tailored to its own economic situation. Interestingly, our result is consistent with the novel finding of Canova, Ciccarelli, and Ortega (2007). They

[35] Strong positive correlations during P6 and P7 occurred during synchronized expansions. However, As shown below, they were driven by the stock market crash in 1987 and its aftermath amid overall expansionary era.

find that the macroeconomic fluctuations among the G-7 countries are more synchronized in recessions than in growth phases. Thus we can conclude statistically that asymmetry in the synchronization of international business cycles is transferable to the cross-country correlations of the bond yields.

(3) Alternation of synchronization and dis-synchronization: P9 and P11

During P9 (from November 1993 to November 2000) and P11 (from January 2002 to November 2002) the behavior of the correlation is quite unique. Before November 1993, the correlation stayed in a positive territory for most of time. Since then, the correlation showed a pronounced sign-switching oscillation around zero except P10. About 50 percent of the correlation estimates was negative during these two periods.

Interestingly, the two economies experienced different evolution during P9 and P11. First, the U.S. economy was in a long expansion ('long boom') since April 1991, which can be characterized by a robust economic growth coupled with very stable inflationary pressures *aka* 'long boom.'[36] In contrast, because of the burst of the bubble in the early 1990s, the Japanese economy was plagued by a prolonged stagnation with sustained deflationary pressures. Ito (2004) characterizes the Japanese economy during this period (from 1993 to 2003) as a *lost decade*.[37] Existing literature on international business cycle provides interesting findings about the co-movements of the U.S. and Japanese economy during P9 and P11. Doyle and Frost (2005), Stock and Watson (2005), Kose, Otrok, and Whiteman (2008) document that the business cycles in Japanese economy became detached from those in other developed countries including the U.S. during P9 and P11. First, Doyle and Frost (2005) and Stock and Watson (2005) find that the correlation of the GDP growths of the U.S. and Japan remained very low and it dropped even to a negative territory. Second, they state that the volatility of the GDP growth of the U.S. economy moderated substantially, while the volatility of the Japanese output growth has increased due to domestic shocks. Third, Kose, Otrok, and Whiteman (2008) find that the growth rate of Japanese output has been less affected by their G-7 common factors during P9, and thus the business cycle of Japan became dis-synchronized from that of other countries such as the U.S. Thus, a decrease in the degree of business cycle synchronization between the U.S. and Japanese economy seemed to be an important cause for the weaker co-movements of the two bond yields.[38]

As a result, monetary policies of the two countries undertook different courses during P9 and P11, which are illustrated in Figure 20.[39]. First, monetary policy of the Fed was largely

[36] See Taylor (1998).

[37] See also Hayashi and Prescott (2002), Ferguson (2005), Mishkin (2008), and Hamada and Okada (2009).

[38] Reflecting the prolonged low synchronicity of the business cycles, market participants' confidence about the current state of the economy and expectation for future also seemed to be detached during the 1990s [See IMF (2001)].

[39] Figure 20 shows the relationship between the policy interest rates of the Fed (Federal Reserve System) and the BOJ (Bank of Japan) and the correlation. Throughout this paper, we measure the stance of the monetary policy in terms of the changes in policy interest rates for both the Fed and the BOJ except the quantitative easing period of the BOJ, which started in March 2001. It is well known that the Fed's policy instrument was not the Federal funds rate targeting between 1979 and 1982. The Fed was targeting nonborrowed bank reserves. However, as presented by Cook (1989) and Goodfriend (1993), the Fed's policy stance can be inferred from the changes in the federal fund rate even that period. The policy interest rate of the BOJ was the official discount rate, the rate at which the BOJ rediscounts bills or extends loans to financial institutions, before September 1995. Since then, the policy instrument of the BOJ shifted from the official discount rate to market operations for controlling the

tightening.[40] In contrast, the BOJ conducted prolonged monetary easing including the zero interest rate policy adopted in February 1999, and the quantitative easing policy from March 2001 to 2006. Thus, the decoupling of monetary policies for such a prolonged time induced weaker correlations between the two yields. However, from time to time, the Fed conducted a mild monetary easing during P9: for example, between the second half of 1995 and the early 1996 in response to a brief and shallow economic slowdown [See Hester (2008)] and from September to November 1998 in the aftermath of the Russian debt default. As such, during P9, the correlation altered its sign depending on the policy stance of the Fed amid persistently dovish monetary policy of the BOJ. This explains the sign-switching oscillation of correlations. In contrast, during P11, both countries' monetary policy was relatively dormant after consecutive rate cuts in the U.S. and Japan during P10 to cope with the IT burble burst. The U.S. economy was struggling to escape from the trough on the back of aggressive monetary easing while Japan's economy was temporarily showing a sign of resurrection on the back of unexpectedly strong export to China. So both countries' economy lost a strong directional trend with a sidewalk move, which results in an oscillation of correlation around zero during P11.

6.2 Non-business-cycle related monetary policies & markets' expectation: P4, P5, P6, P7 and P8

Because we are exploiting the shorter-end of the yield curves of the U.S. and Japan, changes in the monetary policies of the two countries and market participants' expectation on them are expected to be primary drivers of the correlation.

In the previous subsection, we explored concatenation between the two countries' business cycles and conditional correlations of yields. Even in such a case, the two countries' monetary policies play a climatical role as a 'go-between.' However, we directly match business cycles to the correlations mainly because the business cycles are more fundamental.

In this subsection, we consider a change in the two countries' monetary policies and/or the market's expectation about them which are not driven directly by business cycles but by other factors such as international politics and financial crashes. More specifically, coordination and discoordination in monetary policies of the two countries are examined as the primary driving forces behind the correlation between the two bond yields. Periods from P4 to P8 correspond to such episodes. During this period, the correlations showed a roller-coaster move on the back of a sequential advent of historical events such as the Plaza Accord (1985), the Louvre Accord (1987) and the '87 stock market crash (1987). Here we explore each period in a chronological order.

(1) The Plaza Accord: P4

The episode of the period 4 (October 29, 1985) presents a good example of an ephemeral shock to the correlation of the yields. As presented in Figure 18, the preferred habitat of the correlation

uncollateralized call rate, and between September 1985 and August 1998, the official discount rate served as the upper limit of the uncollateralized call rate. From the monetary policy meeting (MPM) in September 1998, the BOJ began to set the target level for the uncollateralized call rate. See Ito and Mishkin (2004), Ito (2009), and Minutes of the MPMs on January 13, 1998 and on September 9, 1998 for the changes in the policy interest rate of the BOJ. MPMs of the BOJ can be found at www.boj.or.jp/en/type/release/teiki/giji.

[40] The Fed conducted a mild monetary easing between the second half of 1995 and the early 1996 in response to a brief and shallow economic slowdown [See Hester (2008)]. Similarly, the Fed reduced the Federal funds rates from September to November 1998 in the aftermath of the Russian debt default.

before P4 was a positive territory. The correlation abruptly dropped to a negative territory in the period 4, and then immediately returned to a positive territory. More specifically, the correlation stayed in a negative region with its estimated value of -27 percent for only October 29, 1985.

Interestingly, such a drastic move in P4 occurred about a month later of the Plaza Accord (September 22, 1985), in which the finance ministers and central bank governors of the G-5 countries had agreed to encourage the depreciation of the dollar along with other policy actions to adjust external imbalances. According to Hamada and Okada (2009) and Obstfeld (2009), one of the policy actions, through which Japan supported the Accord, was the BOJ (Bank of Japan)'s rate hike.[41] In our sample, the Euroyen yield showed a sharp increase from 6.28 percent in September 17 to 7.24 percent in October 29 while the Eurodollar yield showed a mild decrease from 8.26 percent to 8.14 percent. Thus, the ephemeral change of the correlation in P4 was largely driven by the opposite directional moves of the Eurodollar and Euroyen yields as suggested by the Plaza Accord.

(2) The Louvre Accord: P5

During P5 (April 1987-September 1987), the Eurodollar and Euroyen yield were negatively correlated. The correlation remained in negative territory ranging from -30 percent to -10 percent except for a slightly positive value in May. Thus we consider this period as one of the negative correlation regime.

From the spring of 1986 to the winter of 1987, both the Fed and the BOJ adopted quite expansionary monetary policy. This concerted monetary easing was caused in part by the internationally coordinated efforts to lower smoothly the foreign exchange rate value of the U.S. Dollar and by the dramatic drop in oil prices.[42] First, the Federal funds rate was cut from 7.75 percent in January 1986 to 5.875 percent in August 1986, and remained at the essentially same level until March 1987. Second, the Japanese economy entered into a mild recession from June 1985 to November 1986 caused by the sharp appreciation of the yen.[43] The BOJ promoted monetary easing to counter the deflationary impact of the rapid appreciation of the yen.[44] The discount rate was cut from 5 percent in January 1986 to 3 percent in January 1987.

However, the monetary policies of the Fed and the BOJ began to decouple from the Spring of 1987 when the dollar was in a heavy downward pressure. In the Louvre Accord (February 22, 1987), the finance ministrations and central bank governors of the major developed countries including the U.S. and Japan agreed to cooperate to stabilize the foreign exchange rates. The Fed supported internationally coordinated efforts by raising the Federal funds rate by 50 basis points in April and May when the dollar fell sharply against other key currencies.[45] Rising inflationary pressures largely caused by the sharp depreciation of the dollar was another reason

[41]Hamada and Okada (2009) note that the BOJ raised the call rate from October to December 1985. It seems that those actions of the BOJ was temporal. More sustained effects of the Plaza Accord on the BOJ's monetary policy are discussed later.

[42]See Board of Governors (1986, 1987a) and Okina, Shirakawa, and Shiratsuka (2001)

[43]See Ueda (1991) and Okina, Shirakawa, and Shiratsuka (2001). The reference dates of the recession are based on the official business cycle dating of ESRI.

[44]See Ueda (1991).

[45]See Board of Governors (1987b) and Mussa (1994).

for the rate hikes in April and May.[46] As we can see in the first plot of Figure 21, the Eurodollar yield rose considerably during April and May reflecting the rate hikes of the Fed.[47] In contrast, the BOJ cut its discount rate by 50 basis points at February 23, 1987 to support the Louvre Accord.[48] Although the discount rate of the BOJ was kept unchanged, the Euroyen yield gradually fall until May as presented in the second plot of Figure 21. Because the rate cut in February was primarily intended to stabilize the appreciation of the yen, it seems that the rapid drop in the dollar against the yen in April and May led market participants to expect continuation of the BOJ's monetary easing.[49] As a result, the negative correlation between the Eurodollar and Euroyen yields in April and May of P5 were largely driven by the monetary tightening of the Fed and market participants' expectation of the protraction of the BOJ's easing policy.

From June to August, both the Fed and the BOJ were calm.[50] However, the two bond yields showed a different picture. The first plot of Figure 21 suggests that the Eurodollar yield fell substantially from June to August. In contrast, the Euroyen yield rose gradually as presented in the second plot of Figure 21. From June to August, the dollar recovered slightly against other key currencies, including the yen, and then stabilized in the foreign exchange markets. Board of Governors (1987b) notes that the monetary tightening of the Fed in April and May together with monetary easing moves of major developed countries, such as Japan, helped to stabilize the dollar and calm inflation fears, contributing decline in the U.S. interest rates. The rise of the Euroyen yield can be interpreted in a similar vein. Because the BOJ's rate cut in February was largely to act against the rapid appreciation of the yen, the stabilization of the yen seemed to soften market participants' expectation on monetary easing.[51] As such, the negative correlation between the Eurodollar and Euroyen yields between July and August can be viewed as a reflection of the decoupling in the market participants' expectation about the monetary policies.

After the tranquil period from June to August, the Fed restarted rate hikes from the beginning of September to act against increased concern about the inflation.[52] Reflecting the Fed's rate hikes, the Eurodollar yield rose substantially. It increased from 7.36 percent in September 22, which is the last date the period 5, to 8.74 percent in October 13, which is the single date between the period 5 and the period 6. Although there was no change in the BOJ's discount rate, the Euroyen yield showed a considerable increase from 4.48 percent in September 13 to 5.03 percent in October 13. According to Ueda (2000), Okina, Shirakawa, and Shiratsuka (2001), and

[46]See Board of Governors (1987b) and Greenspan (2004).

[47]The rise of the U.S. bond yields in April and May was addressed in the FOMC (Federal Open Market Committee) meeting held on May 19. The Record of Policy Actions states that the downward pressure on the dollar created concern among market participants about the prospects for inflation and the response of monetary policy, and this concern seemed to contribute to the rise of bond yields in April and May. The Record of Policy Actions for the FOMC meeting held on May 19 can be found at www.federalreserve.gov/monetarypolicy/fomchistorical1987.htm.

[48]See Okina, Shirakawa, and Shiratsuka (2001).

[49]According to Shinotsuka (2000), public opinion of Japan in the Spring of 1987 called for continuing monetary easing to weaken the appreciation of the yen.

[50]Strictly speaking, we mean June through mid-August since we have only one correlation observation in August 1987 (August 11). Accordingly, the following analysis are applied from June to mid-August.

[51]The rise of the Japanese interest rates between June and August was addressed in the *Record of Policy Actions* of the FOMC meetings held on July 7 and August 18. The FOMC stated that the increase in the Japanese interest rates was attributed to signs of stronger economic activity. In fact, as noted by Ueda (1991), in spite of the sharp appreciation of the yen, the Japanese economy recovered from the recession fairy quickly. According the ESRI business cycle, the Japanese economy bottomed out as of the Spring of 1987.

[52]See Mussa (1994) and Goodfriend (2002)

Shiratsuka (2005), there seemed to be a change in the stance of the BOJ from easing to tightening between September and October. In particular, Okina, Shirakawa, and Shiratsuka (2001) point out:

In view of the prospective hike in the official discount rate, the BOJ took the first concrete step to change its monetary easing stance at the end of August 1987 when it began guiding market interest rates to a higher level. As a result, short-term market rates gradually rose after September and, on October 19, immediately before Black Monday in the United States ...

Long-term interest rate also rose by nearly three percentage points compared with the lowest level, reflecting clear signs of economic recovery, an increase in money supply, and the rebound of commodity prices both domestically and overseas.[53]

Thus, the Eurodollar yield was rising in response to the rate hikes of the Fed and the Euroyen yield was pricing the changes in the BOJ's policy stance. Interestingly, the correlation between the two bond yields increased abruptly from -30 percent in September 22 to 25 percent in October 13.

(3) Stock Market Crash: P6 and P7

P6 (November 1987-May 1988) begins with the impact of the stock market crash in October 19, 1987. In the aftermath of the stock market crash, between October 13 (the single data point between P5 and P6) and November 3 (the first date of P6), the Eurodollar yield fell by 120 basis points, and the Euroyen yield also decreased by 70 basis points. Reflecting those sharp adjustments in the bond yields, the correlation increased sharply from 25 percent in October 13 to 65 percent in November 3. Since then, the correlation stayed in a highly positive territory. During P6, the correlation ranged from 52 percent to 79 percent. That highly positive correlation continued until P7 (August 1988-October 1988). During P7, the correlation ranged from 54 percent to 63 percent. Even in the intermediate dates between P6 and P7, the correlation was around 50 percent.[54]

As discussed above, the stance of both the Fed and the BOJ turned into monetary tightening from October 1987. However, those moves were surprised by the stock market crash.[55] The Fed reduced the Federal funds rate from November 1987 to February 1988 to stabilize highly volatile financial markets and to cushion the effects of the stock market decline on the economy.[56] According to Okina, Shirakawa, and Shiratsuka (2001), the stock market crash led the BOJ to defer the policy actions to guide interest rates to a higher level.[57] Reflecting the coupled moves of the Fed and the BOJ, the correlation between the two bond yields were strongly positive between November 1987 to February 1988.

As of March 1988, the Fed returned to monetary tightening as the economy showed strong growth.[58] Rate hikes continued until February 1989. As presented in the first plot of Figure

[53] As the public relations department of the BOJ confirmed us, the current disclosure system of the BOJ does not yet provide information related to the BOJ's policy actions prior to 1998.

[54] There were three data points between the period 6 and the period 7: May 31, June 21, and July 12, 1987. The estimated correlation during those three data points were 48 percent, 51 percent, and 47 percent, respectively.

[55] See Bernanke and Mishkin (1992), Mussa (1994), Goodfriend (2002), and Greenspan (2004) for the U.S., and see Ueda (2000) and Okina, Shirakawa, and Shiratsuka (2001) for Japan.

[56] See Board of Governors (1988).

[57] Similarly, Ueda (2000) points out that the BOJ showed no clear attempts at tightening after the U.S. stock market crash. At that time, the primary concern of the BOJ was on the decline in foreign stocks.

[58] See Mussa (1994) and Greenspan (2004).

21, the Eurodollar yield kept rising until the end of the period 7 in response to the consecutive hikes. It rose from 6.94 percent in March 29, 1988 to 8.38 percent in September 13, 1988. Although the BOJ's discount rate was still at 2.5 percent, the second plot of the Figure 21 shows that the Euroyen yield stopped declining in April 1989 and started to rise from May 1989. It rose from 4.11 percent in April 19, 1988 to 4.94 percent in September 13, 1988. According to Ueda (1993), the upturn of the Euroyen yield seemed to be caused by market participants' expectation for the BOJ's monetary tightening. In particular, Ueda (1993) notes:

In the summer of 1988, short-term interest rates in open markets, such as the CD rate and Euroyen rates, increased as a result of an expectation of a future tightening of monetary policy.

Thus, the primary drivers of the strong coupling of the Eurodollar and Euroyen yields from the later part of P6 to P7 were the rate hikes of the Fed and the market participants' expectation for the BOJ's tightening monetary policy.

(4) Back on track: P8

After the monetary tightening between March 1988 and February 1989, the Fed began to reduce the interest rate from June 1989 responding to significant slowing of the economic activity.[59] The Federal funds rate was reduced from 9.75 percent in May 1989 to 8.25 percent in December 1989. Pricing the Fed's rate cuts, the Eurodollar yield fell sharply during the period 8 as shown in the first plot of Figure 21. As discussed, the BOJ's discount rate stayed at 2.5 percent since March 1987. At the end of May 1989, the BOJ finally raised its discount rate from 2.5 percent to 3.25 percent to act against increasing inflationary pressures including soaring asset prices.[60] Since then, it was raised to 3.75 percent in December 1989. As presented in the second plot of Figure 21, the Euroyen yield rose reflecting the rate hikes of the BOJ.

6.3 Summary

We classify the SNP conditional correlation estimates into three regimes: strong positive correlation, negative correlation and sign-switching oscillation. 11 separate periods match the operational definitions of those three regimes, which accommodate most of pronounced patterns of time-varying behavior of correlations. Out of 11 periods, five periods (P1, P2, P9, P10, and P11) can be explained by synchronization or dis-synchronization of the two countries' business cycles. More specifically, P1, P2 and P10 (which belong to the strong positive regime) could be elucidated directly by synchronization of the two countries' official recessions; the sign-switching oscillatory behavior observed in P9 and P11 can be explained by mild fluctuations in business cycles which are not officially announced. As a byproduct, we find that coupling in the two countries' interest rates is strongest when both countries fall in economic downturn; decoupling is, in contrast, more likely to occur when both countries are in expansionary mode. Other four periods (P4, P5, P6, P7 and P8) correspond to major upheavals in global exchange rate coordination such as the Plaza Accord and Louvre Accord and cataclysmal financial market crash, which are not directly accounted for by business cycles.

P3 is the only period, which is associated with neither business cycles nor monetary policies. The existing literature suggests that the strong positive correlation then might be induced by

[59]See Mussa (1994) and Greenspan (2004).

[60]See Bernanke and Gertler (1999) and Yamaguchi (2000).

market participants' expectation, but we left this period as 'not explicitly explicable.

In summary, given the fact that 10 out of 11 noteworthy periods designated by the SNP can be backed up by fundamentals, the SNP correlation is economically meaningful. That said, we can accept the SNP correlation estimates as an admissible benchmark for testing the performance of the $\mathbb{IATSM}$s and the TAs in reproducing the time varying correlations.

7 Can the $\mathbb{IATSM}$s and the $\mathbb{IQTSM}$s reproduce the SNP correlations?

In this section, we compare the performances of the five international term structure models in reproducing the projected conditional correlation between the Eurodollar and Euroyen yields.

7.1 $\mathbb{IATSM}$s

Figures 22 and 23 present the conditional correlation implied by the international affine term structure models. It is clear that all of the four affine models have severe difficulty in reproducing the conditional correlation implied by the data.

First, it is clear that $\mathbb{IA}_{1;1,0,0}(3;3,0,0)$ cannot reproduce the correlation implied by the data. The correlation implied by $\mathbb{IA}_{1;1,0,0}(3;3,0,0)$ is much smoother than the projected correlation. As depicted in the plot, this model cannot fit any characteristic of the correlation implied by the data.

Second, $\mathbb{IA}_{2;2,0,0}(3;3,0,0)$, $\mathbb{IA}_{2;2,0,0}(4;2,1,1)$, and $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$ show similar performance. They fail to track the projected correlation for the period between 1990 and 2001. However, they do a somewhat better job in matching the correlation for the 1980s where the preferred habitat of the projected correlation is a positive territory. Particularly, they are able to reproduce the sharp increase in the correlation at the start of P6 and the strong positive correlation during P6 and P7, which are the parts of Regime P.

Third, and more importantly, all of the four affine models cannot generate the sign-switching property of the correlation. The conditional correlation implied by $\mathbb{IA}_{1;1,0,0}(3;3,0,0)$ switches its sign from negative to positive in the early 1980s where the two bond yields showed high positive correlation. Excepting that single case, all of the correlation estimates implied by the four affine models remain in a positive territory. As a result, they cannot reproduce the negative correlation in P4, P5, and P8 (Regime N). Similarly, the sign-switching oscillatory property of the projected correlation observed in P9 and P11 (Regime O) cannot be reproduced by the four affine model.

As shown in Section 3, these four affine models are theoretically capable of generating the sign-switching correlation between the two yields. Thus, it is important to investigate the reason why the models cannot reproduce none of the episodes in Regime N and Regime O. In Section 4.4, we demonstrate that the mechanism for generating the sign-switching correlation between $r_d(t)$ and $r_f(t)$ in the four affine models requires some parametric restrictions. First, the correlation between $r_d(t)$ and $r_f(t)$ implied by $\mathbb{IA}_{1;1,0,0}(3;3,0,0)$ can switch its sign only when the sample path of the square-root common factor, $Y_1(t)$, passes the intercept of the covariance function, $-\frac{\delta_2^d\delta_2^f+\delta_3^d\delta_3^f}{\delta_1^d\delta_1^f+\delta_2^d\delta_2^f\beta_{21}+\delta_3^d\delta_3^f\beta_{31}}$. Inserting the EMM estimates in Table 2 reveals that the estimated value of this intercept is around 20. This implies that the model can generate the sign-switching correlation only when the estimated conditional volatility of each short rate is very high since the estimate for the long-run mean of $Y_1(t)$, θ_1, is about 6.25. This explains why the reprojected correlation implied by $\mathbb{A}_{1;1,0,0}(3;3,0,0)$ changes its sign in the early 1980s. Similarly, $\mathbb{A}_{3;1,1,1}(4;2,1,1)$ is able to induce the sign-switching correlation only when the square-root common factor, $Y_1(t)$, passes the intercept, $-\frac{\delta_4^d\delta_4^f}{\delta_1^d\delta_1^f+\delta_4^d\delta_4^f\beta_{41}}$. Our EMM estimates suggest that the estimated value of the intercept is about -0.36. As a result, $\mathbb{A}_{3;1,1,1}(4;2,1,1)$ cannot generate the sign-switching correlation since the admissibility condition requires that $Y_1(t)$ cannot take negative value. In the case of $\mathbb{A}_{2;2,0,0}(3;3,0,0)$, the sign-switching correlation can be induced when the square-root common factor, $Y_1(t)$, passes the intercept of the conditional covariance function, $-\frac{\delta_3^d\delta_3^f+\left(\delta_2^d\delta_2^f+\delta_3^d\delta_3^f\beta_{32}\right)Y_2(t)}{\delta_1^d\delta_1^f+\delta_3^d\delta_3^f\beta_{31}}$. The EMM estimates suggest that the correlation implied by $\mathbb{A}_{2;2,0,0}(3;3,0,0)$ is positive if $Y_1(t)$ is larger than $-0.09+2.93Y_2(t)$, and vice versa. However, this imposes a severe restriction on the dynamics of $Y_1(t)$ since the estimated value of θ_1, 1.61, is about half of the estimated value of θ_2, 3.26. Finally, in the case of $\mathbb{A}_{2;2,0,0}(4;2,1,1)$, the minimum necessary condition for generating the sign-switching correlation is either $\delta_1^d\delta_1^f<0$ or $\delta_2^d\delta_2^f<0$. However, the EMM estimates indicate that the estimated sensitivities of $r_d(t)$ and $r_f(t)$ to the two square-root common factors, $Y_1(t)$ and $Y_2(t)$, are strictly positive. Thus, $\mathbb{A}_{2;2,0,0}(4;2,1,1)$ is incapable of generating the sign-switching correlation between $r_d(t)$ and $r_f(t)$.

Our results suggest that reproducing the sign-switching behavior of the correlation imposes severe restrictions to $\mathbb{ATSM}$s in matching other characteristics of the data. As discussed, all of the affine models are quite well in fitting the conditional first moments of the two yields. $\mathbb{A}_{2;2,0,0}(3;3,0,0)$, $\mathbb{A}_{2;2,0,0}(4;2,1,1)$, and $\mathbb{A}_{3;1,1,1}(4;2,1,1)$ are much better in tracking the conditional volatility of both the two yields and the USD/JPY return than in matching the conditional correlation between the two yields.

7.2 $\mathbb{QTSM}$

Figure 23 compares the conditional correlation implied by $\mathbb{Q}(3;1,1,1)$ to that implied by the data. Although $\mathbb{Q}(3;1,1,1)$ is much better than the four affine models in capturing the time-varying correlation, its performance in reproducing the sign-switching property of the correlation is limited.

First, among the episodes in Regime P, $\mathbb{Q}(3;1,1,1)$ is able to reasonably describe the high positive correlation in P1, P3, P6, P7, and P10. Particularly, it can reproduce the abrupt increase in the correlation at the beginning of P6 and P10. In the case of P2, $\mathbb{Q}(3;1,1,1)$ can reproduce the shape of the correlation. However, it considerably underestimates the level of the correlation.

Second, $\mathbb{Q}(3;1,1,1)$ fails to reproduce the negative correlation of Regime N. It cannot reproduce the ephemeral negative correlation observed in P4. The correlation implied by $\mathbb{Q}(3;1,1,1)$ remains in a positive territory during P5 where the SNP projected correlation stayed at the negative domain. Similarly, $\mathbb{Q}(3;1,1,1)$ fails to induce the negative correlation in P8. Thus, our results indicate that $\mathbb{Q}(3;1,1,1)$ is not able to match the correlation dynamics in Regime N.

Third, $\mathbb{IQ}(3;1,1,1)$ shows a better performance for the episodes in Regime O. $\mathbb{IQ}(3;1,1,1)$ is able to reproduce the long swing in the correlation, and thus able to match the switches in the sign of the correlation in P9 and P11. However, it cannot reproduce the ephemerally sign-switching behavior of the correlation.

In summary, our reprojection results for both $\mathbb{IATSM}$s and $\mathbb{IQTSM}$ suggest that the models experience considerable difficulty in reproducing the conditional correlation between the two bond yields compare to other characteristics of the data. In particular, although $\mathbb{IQ}(3;1,1,1)$ shows surprising performance in matching the conditional moments of the data, its performance is not satisfactory in reproducing the sign-switching behavior of the correlation between the two yields.

8 Conclusion

This paper investigates the empirical property of the sign-switching correlations of cross-country interest rates and explores whether the international term structure models are able to reproduce such behavior theoretically and empirically. We can summarize our major findings as following:

- The $\mathbb{IATSM}$s cannot simultaneously allow for sign-switching correlations and guarantee positivity of the nominal interest rates. When the negativity of the interest rate is allowed, a subset of $\mathbb{IATSM}$s are found to generate the sign-switching correlations *theoretically*. However, even such $\mathbb{IATSM}$s still suffer from a theoretical drawback which hampers their performance in reproducing the sign-switching correlations. In order for the $\mathbb{IATSM}$s to generate negative cross-country correlations, a key necessary condition is the opposite signs of the sensitivities of the interest rates to the common factors. Among those common factors that each country's interest rate responds in the opposite directions to, the Gaussian factors have less contribution to engendering the sign-switching pattern in cross-country correlations due to their homoskedastisticity. As such, it is essential that each country reacts in the opposite directions to the square-root common factors to allow for negative correlations *and* heteroskedasticity. However this means that the product of those square-root common factors and one particular country's sensitivities to them are always negative in determining that country's nominal interest rate. Consequently, to generate the large amount of negative correlations, one of two countries' interest rate is more likely to fall into a negative domain, which is, of course, at odds with data. To make it worse, admissibility of the $\mathbb{IATSM}$s requires nonnegative correlations among the square-root common factors. As a result, if a particular square-root common factor drives negative correlations, other common factors are likely to show the same behavior. That limits the flexibility of the $\mathbb{IATSM}$s in reproducing the sign-switching behavior of correlations.
- In contrast, the $\mathbb{IQTSM}$s have potential to overcome the aforementioned limitations of the $\mathbb{IATSM}$s. The $\mathbb{IQTSM}$s can theoretical allow for the sign-switching correlations and guarantee the positivity of the nominal interest rate. In addition, unlike the $\mathbb{IATSM}$s, the $\mathbb{IQTSM}$s does not rely solely upon the opposite signs of sensitivities to the common factors to generate the sign-switching correlations. The Gaussian factors themselves, which could be either positively or negatively correlated, are sign-switching, thereby enabling the signs of cross-country correlations switching over time. A nice byproduct of that feature is that local factors, which should exist to validate the spanning enhancement of globalizing a

fixed income portfolio, are also able to contribute to having the correlation sign-switching. Therefore, every single factor contributes to generating the sign-switching correlations and heteroskedasticity without violating the positivity of the nominal interest rate.

- The SNP estimates of time-varying correlations among interest rates are overall well supported by historical occurrences in the U.S. and Japan. Most of extreme values of correlations and their abrupt changes in signs are found to be driven by synchronization and desynchronization of business cycles (especially recessions), monetary policies, expectation of monetary policy changes and ephemeral market crashes. This finding validates the quality of the SNP estimates of correlations.

- Even the subfamilies of the $\mathbb{IATSM}$s selected to be able to theoretically generate the sign-switching correlations fail to reproduce them empirically. The limited flexibility of the $\mathbb{IATSM}$s in characterizing the dynamics of cross-country term structures hampers its performance. The $\mathbb{IATSM}$s sacrifice capturing the time-varying correlations, let alone their sign-switching behavior in order to fit other features in dynamics, especially the levels of interest rates.

- The $\mathbb{IQTSM}$s are found to perform well in reproducing the empirically observed time-varying behavior of the correlations, especially swing moves. However, the models show poor performance in matching the ephemeral shocks to correlations.

Our results suggest a couple of future research issues. First, a more full-fledged empirical study on a broader spectrum of tenors would be desired. In the current paper, we focus on the shorter-end of yield curve so that monetary policies and market's expectation about them are the primary driving forces behind the time-varying correlations. However, correlations in its belly or longer end might be more driven by asset substitution effects and demand/supply shocks. Second, given that the SNP estimates of time-varying correlations are economically meaningful, a more in-depth exploration about coordination and discoordination of global monetary policies would be an interesting topic. Finally, we need a more sophisticated international term structure which is able to reproduce ephemeral shocks to correlations would be desired. Introducing jumps to the dynamics of cross-country term structure might be such a candidate.

Appendix

A. International interest rate data

This appendix describes in detail the international interest rate data used for Figure 1. For the cross-country interest rate correlations of U.S. and each of two developed countries (Germany and France), we utilize one-year Eurocurrency interest rate data, which are taken from the Financial Times and are provided by Datastream. These sample cover the period January 22, 1980-December 30, 2008 for both U.S.-Germany and U.S.-France. For the correlations of U.S. and each of two emerging countries (Brazil and Poland), we employ Bloomberg-constructed two-year generic government bond interest rate data. These sample cover the period April 18, 2000-December 30, 2008 for U.S.-Brazil, and March 23, 1999-December 30, 2008 for U.S.-Poland.

The reasons for our choice of different dataset and different maturities are as follows. First, there is no Eurocurrency interest rate available for Brazil and Poland. Second, we do not use the Bloomberg-constructed government bond data for country pairs of U.S. and the two developed countries because this dataset does not cover the period of 1980s. The starting dates of the one-year Bloomberg generic government bond data for Germany and France are January 31, 1995 and June 30, 1989, respectively. Third, we use two-year maturity for the correlations among U.S. and each of the two emerging countries because the Bloomberg-constructed one-year U.S. generic government bond interest rate data includes missing values between August 20, 2001 and June 30, 2008. To alleviate nonsynchronous trading issue, we sample interest rates at a bi-weekly frequency. Further, we exploit Tuesday-to-Tuesday data to void any end-of-the week effect.

The dynamics of the correlation is estimated from bivariate BEKK-GARCH model of Engle and Kroner (1995), wherein the mean dynamics is captured by using the VAR function. In the estimation, we use the bi-weekly changes of the interest rate data.

B. Parametric restrictions on $\mathbb{IA}_{m;m_c,m_d,m_f}(N; N_c, N_d, N_f)$

As shown by Dai and Singleton (2000), the following restrictions are required to ensure the admissibility of $\mathbb{IA}_{m;m_c,m_d,m_f}(N; N_c, N_d, N_f)$.

$$
\begin{aligned}
&\left[K^{B_c B_c}\right]_{ij} \leq 0,\ 1 \leq i \neq j \leq m_c,\\
&\left[K^{B_d B_d}\right]_{ij} \leq 0,\ 1 \leq i \neq j \leq m_d,\\
&\left[K^{B_f B_f}\right]_{ij} \leq 0,\ 1 \leq i \neq j \leq m_f,\\
&\left[K^{B_d B_c}\right]_{ij} \leq 0,\ 1 \leq i \leq m_d,\ 1 \leq j \leq m_c,\\
&\left[K^{B_f B_c}\right]_{ij} \leq 0,\ 1 \leq i \leq m_f,\ 1 \leq j \leq m_c\\
&\left[K^{B_c B_c}\Theta^{B_c}\right]_{i} > 0,\ 1 \leq i \leq m\\
&\left[K^{B_d B_c}\Theta^{B_c} + K^{B_d B_d}\Theta^{B_d}\right]_{i} > 0,\ 1 \leq i \leq m_d\\
&\left[K^{f_d B_c}\Theta^{B_c} + K^{B_f B_f}\Theta^{B_f}\right]_{i} > 0,\ 1 \leq i \leq m_f\\
&\left[\Theta^{B_c}\right]_{i} \geq 0,\ 1 \leq i \leq m_c,\\
&\left[\Theta^{B_d}\right]_{i} \geq 0,\ 1 \leq i \leq m_d,
\end{aligned}
$$

$$
\begin{aligned}
&\left[\Theta^{B_f}\right]_i \geq 0,\ 1 \leq i \leq m_f,\\
&\left[\mathbf{B}^{B_c D_c}\right]_{ij} \geq 0,\ 1 \leq i \leq m_c,\ 1 \leq j \leq N_c - m_c,\\
&\left[\mathbf{B}^{B_c D_d}\right]_{ij} \geq 0,\ 1 \leq i \leq m_c,\ 1 \leq j \leq N_d - m_d,\\
&\left[\mathbf{B}^{B_c D_f}\right]_{ij} \geq 0,\ 1 \leq i \leq m_c,\ 1 \leq j \leq N_f - m_f,\\
&\left[\mathbf{B}^{B_d D_d}\right]_{ij} \geq 0,\ 1 \leq i \leq m_d,\ 1 \leq j \leq N_d - m_d,\\
&\left[\mathbf{B}^{B_f D_f}\right]_{ij} \geq 0,\ 1 \leq i \leq m_f,\ 1 \leq j \leq N_f - m_f.
\end{aligned}
$$

We impose the following restrictions to identify the signs of Gaussian factors, $Y^{D_c}(t)$, $Y^{D_d}(t)$, and $Y^{D_f}(t)$.

$$
\begin{aligned}
&\left[\delta^d_{y^{D_c}}\right]_i \geq 0,\ 1 \leq i \leq N_c - m_c,\\
&\left[\delta^d_{y^{D_d}}\right]_i \geq 0,\ 1 \leq i \leq N_d - m_d,\\
&\left[\delta^f_{y^{D_f}}\right]_i \geq 0,\ 1 \leq i \leq N_f - m_f.
\end{aligned}
$$

C. Proof of Part (a) of Condition 1

We prove that the subfamilies of $\mathbb{A}_{m;m_c,m_d,m_f}(N; N_c, N_d, N_f)$ with $m_c = 1$ and $N_c - m_c = 0$ cannot generate the sign-switching correlation of $r_d(t)$ and $r_f(t)$. In equation (24), equating $\delta^d_{y^{D_c}}$ and $\delta^f_{y^{D_c}}$ to zero yields

$$
\mathrm{Cov}_{df}(t) = \sum_{i=1}^{m_c} \left[\delta^d_{y^{B_c}} \circ \delta^f_{y^{B_c}} \circ Y^{B_c}(t)\right]_i .
$$

Therefore, any subfamily of $IA_{m;m_c,m_b,m_f}(N; m_c, N_d, N_f)$ with $m_c = 1$ and $N_c - m_c = 0$ is incapable of generating the sign-switching correlation.

D. Proof of Proposition 1

Applying $N_c = m_c$, $N_d = m_d$, and $N_f = m_f$ to our canonical model of $\mathbb{A}_{m;m_c,m_d,m_f}(N; N_c, N_d, N_f)$ results in the correlated square-root factor models, wherein

$$
\begin{aligned}
r_d(t) &= \delta^d_0 + {\delta^d_{y^{B_c}}}' Y^{B_c}(t) + {\delta^d_{y^{B_d}}}' Y^{B_d}(t),\\
r_f(t) &= \delta^f_0 + {\delta^f_{y^{B_c}}}' Y^{B_c}(t) + {\delta^f_{y^{B_f}}}' Y^{B_f}(t),
\end{aligned}
$$

and

$$
\begin{aligned}
dY^{B_c}(t) &= K^{B_c B_c}\left(\Theta^{B_c} - Y^{B_c}(t)\right) dt + \sqrt{Y^{B_c}(t)} dW^{B_c}(t),\\
dY^{B_d}(t) &= \left[K^{B_d B_c}\left(\Theta^{B_c} - Y^{B_c}(t)\right) + K^{B_d B_d}\left(\Theta^{B_d} - Y^{B_d}(t)\right)\right] dt + \sqrt{Y^{B_d}(t)} dW^{B_d}(t),\\
dY^{B_f}(t) &= \left[K^{B_f B_c}\left(\Theta^{B_c} - Y^{B_c}(t)\right) + K^{B_f B_f}\left(\Theta^{B_f} - Y^{B_f}(t)\right)\right] dt + \sqrt{Y^{B_f}(t)} dW^{B_f}(t).
\end{aligned}
$$

Applying Ito's lemma leads to the following SDEs of the interest rates:

$$\begin{aligned} dr_d(t) &= \left[\left(\delta^d_{y^{B_c}}{}' K^{B_c B_c} + \delta^d_{y^{B_d}}{}' K^{B_d B_c}\right)\left(\Theta^{B_c} - Y^{B_c}(t)\right) + \delta^d_{y^{B_d}}{}'\left(\Theta^{B_d} - Y^{B_d}(t)\right)\right] dt \\ &+ \delta^d_{y^{B_c}}{}'\sqrt{Y^{B_c}(t)}dW^{B_c}(t) + \delta^d_{y^{B_d}}{}'\sqrt{Y^{B_d}(t)}dW^{B_d}(t), \end{aligned}$$

$$\begin{aligned} dr_f(t) &= \left[\left(\delta^f_{y^{B_c}}{}' K^{B_c B_c} + \delta^f_{y^{B_f}}{}' K^{B_f B_c}\right)\left(\Theta^{B_c} - Y^{B_c}(t)\right) + \delta^f_{y^{B_f}}{}'\left(\Theta^{B_f} - Y^{B_f}(t)\right)\right] dt \\ &+ \delta^f_{y^{B_c}}{}'\sqrt{Y^{B_c}(t)}dW^{B_c}(t) + \delta^f_{y^{B_f}}{}'\sqrt{Y^{B_f}(t)}dW^{B_f}(t). \end{aligned}$$

Thus the correlation coefficient of the two interest rates can be written as:

$$\text{Corr}_{df}(t) = \frac{\text{Cov}_{df}(t)}{\sqrt{\text{Var}_d(t)}\sqrt{\text{Var}_f(t)}}, \tag{75}$$

where

$$\begin{aligned} \text{Cov}_{df}(t) &= \sum_{i=1}^{m_c}\left[\delta^d_{y^{B_c}} \circ \delta^f_{y^{B_c}} \circ Y^{B_c}(t)\right]_i, \\ \text{Var}_d(t) &= \sum_{i=1}^{m_c}\left[\delta^d_{y^{B_c}} \circ \delta^d_{y^{B_c}} \circ Y^{B_c}(t)\right]_i + \sum_{i=1}^{m_d}\left[\delta^d_{y^{B_d}} \circ \delta^d_{y^{B_d}} \circ Y^{B_d}(t)\right]_i, \\ \text{Var}_f(t) &= \sum_{i=1}^{m_c}\left[\delta^f_{y^{B_c}} \circ \delta^f_{y^{B_c}} \circ Y^{B_c}(t)\right]_i + \sum_{i=1}^{m_f}\left[\delta^f_{y^{B_f}} \circ \delta^f_{y^{B_f}} \circ Y^{B_f}(t)\right]_i. \end{aligned}$$

From equation (75), a necessary condition to accommodate the sign-switching correlation is that the signs of some elements of $(\delta^d_1\delta^f_1, \delta^d_2\delta^f_2, \cdots, \delta^d_{m_c}\delta^f_{m_c})$ are negative. This condition leads to a positive probability of generating negative interest rates because $Y^{B_c}(t)$ is not bounded from above. Therefore the subfamily models in $IA_{m;m_c,m_b,m_f}(N; m_c, m_d, m_f)$ cannot accommodate the sign-switching property of the correlation without violating the positivity of the interest rates.

E. Proof of Proposition 2

Equating m_c, m_d, and m_f to zero in our canonical model of $\mathbb{IA}_{m;m_c,m_d,m_f}(N; N_c, N_d, N_f)$ leads to the following Gaussian factor models:

$$\begin{aligned} r_d(t) &= \delta^d_0 + \delta^d_{y^{D_c}}{}' Y^{D_c}(t) + \delta^d_{y^{D_d}}{}' Y^{D_d}(t), \\ r_f(t) &= \delta^f_0 + \delta^f_{y^{D_c}}{}' Y^{D_c}(t) + \delta^f_{y^{D_f}}{}' Y^{D_f}(t), \end{aligned}$$

and

$$\begin{aligned} dY^{D_c}(t) &= -K^{D_c D_c} Y^{D_c}(t)dt + dW^{D_c}(t), \\ dY^{D_d}(t) &= \left(-K^{D_d D_c} Y^{D_c}(t) - K^{D_d D_d} Y^{D_d}(t)\right) dt + dW^{D_d}(t), \\ dY^{D_f}(t) &= \left(-K^{D_f D_c} Y^{D_c}(t) - K^{D_f D_f} Y^{D_f}(t)\right) dt + dW^{D_f}(t). \end{aligned}$$

By Ito's lemma, the SDEs of the interest rates are represented as:

$$\begin{aligned} dr_d(t) &= \left[-\left(\delta^d_{y^{D_c}}{}'K^{D_cD_c} + \delta^d_{y^{D_d}}{}'K^{D_dD_c}\right)Y^{D_c}(t) - \delta^d_{y^{D_d}}{}'K^{D_dD_d}Y^{D_d}(t)\right]dt \\ &+ \delta^d_{y^{D_c}}{}'dW^{D_c}(t) + \delta^d_{y^{D_d}}{}'dW^{D_d}(t), \end{aligned}$$

$$\begin{aligned} dr_f(t) &= \left[-\left(\delta^f_{y^{D_c}}{}'K^{D_cD_c} + \delta^f_{y^{D_f}}{}'K^{D_fD_c}\right)Y^{D_c}(t) - \delta^f_{y^{D_f}}{}'K^{D_fD_f}Y^{D_f}(t)\right]dt \\ &+ \delta^f_{y^{D_c}}{}'dW^{D_c}(t) + \delta^f_{y^{D_f}}{}'dW^{D_f}(t). \end{aligned}$$

Thus the correlation coefficient of the two interest rates can be written as:

$$\mathrm{Corr}_{df}(t) = \frac{\mathrm{Cov}_{df}(t)}{\sqrt{\mathrm{Var}_d(t)}\sqrt{\mathrm{Var}_f(t)}}, \tag{76}$$

where

$$\begin{aligned} \mathrm{Cov}_{df}(t) &= \sum_{i=1}^{N_c}\left[\delta^d_{y^{D_c}} \circ \delta^f_{y^{D_c}}\right]_i, \\ \mathrm{Var}_d(t) &= \sum_{i=1}^{N_c}\left[\delta^d_{y^{D_c}} \circ \delta^d_{y^{D_c}}\right]_i + \sum_{i=1}^{m_d}\left[\delta^d_{y^{D_d}} \circ \delta^d_{y^{D_d}}\right]_i, \\ \mathrm{Var}_f(t) &= \sum_{i=1}^{N_c}\left[\delta^f_{y^{D_c}} \circ \delta^f_{y^{D_c}}\right]_i + \sum_{i=1}^{m_f}\left[\delta^f_{y^{D_f}} \circ \delta^f_{y^{D_f}}\right]_i, \end{aligned}$$

Equation (76) states that any subfamily in $IA_{0;0,0,0}(N; N_c, N_d, N_f)$ can generate only homoskedastic correlation of $r_d(t)$ and $r_f(t)$, which can be either positive or negative.

F. Proof of Proposition 3

An implication of part (a) of Condition 1 is that we need two or more common factors to model the sign-switching correlation of $r_d(t)$ and $r_f(t)$. Therefore, any three-factor $\mathbb{IATSM}$ with both common and local factors cannot accommodate the sign-switching property of the correlation.

Table 1 states that there are eight subfamilies of three-factor $\mathbb{IATSM}$s. According to Proposition 1, both $IA_{3;3,0,0}(3; 3, 0, 0)$ and $IA_{3;1,1,1}(3; 1, 1, 1)$ cannot generate the sign-switching correlation without violating the positivity of the interest rates. Proposition 3 implies that neither $IA_{0;0,0,0}(3; 3, 0, 0)$ nor $IA_{0;0,0,0}(3; 1, 1, 1)$ is able to accommodate the sign-switching correlation. Among the remaining four models, both $\mathbb{IA}_{1;1,0,0}(3; 3, 0, 0)$ and $\mathbb{IA}_{2;2,0,0}(3; 3, 0, 0)$ are theoretically capable of inducing the sign-switching correlation as shown in Section 3. However, neither model can accommodate the country-specific movements of $r_d(t)$ and $r_f(t)$ because they include common factors only. In this appendix, we prove that both models $IA_{1;1,0,0}(3; 1, 1, 1)$ and $IA_{2;0,1,1}(3; 1, 1, 1)$, which are able to accommodate the country-specific interest rate dynamics, are incapable of generating the sign-switching correlation.

$\underline{IA_{1;1,0,0}(3; 1, 1, 1)}$

This model is characterized by one square-root common factor, $Y_1(t)$, one Gaussian domestic local factor, $Y_2(t)$, and one Gaussian foreign local factor, $Y_3(t)$. The two nominal interest rates are given as:

$$\begin{aligned} r_d(t) &= \delta_0^d + \delta_1^d Y_1(t) + \delta_2^d Y_2(t), \\ r_f(t) &= \delta_0^f + \delta_1^f Y_1(t) + \delta_3^f Y_3(t). \end{aligned}$$

The SDEs of the state variables are

$$\begin{aligned} d\begin{pmatrix} Y_1(t) \\ Y_2(t) \\ Y_3(t) \end{pmatrix} &= \begin{pmatrix} \kappa_{11} & 0 & 0 \\ \kappa_{21} & \kappa_{22} & 0 \\ \kappa_{31} & 0 & \kappa_{33} \end{pmatrix} \begin{pmatrix} \theta_1 & - & Y_1(t) \\ & - & Y_2(t) \\ & - & Y_3(t) \end{pmatrix} dt \\ &+ \begin{pmatrix} \sqrt{Y_1(t)} & 0 & 0 \\ 0 & \sqrt{1+\beta_{21}Y_1(t)} & 0 \\ 0 & 0 & \sqrt{1+\beta_{31}Y_1(t)} \end{pmatrix} dW(t). \end{aligned}$$

From Ito's lemma, the instantaneous correlation of the interest rates can be represented as:

$$\mathrm{Corr}_{df}(t) = \frac{\mathrm{Cov}_{df}(t)}{\sqrt{\mathrm{Var}_d(t)}\sqrt{\mathrm{Var}_f(t)}}, \tag{77}$$

where

$$\begin{aligned} \mathrm{Var}_d(t) &= {\delta_1^d}^2 Y_1(t) + {\delta_2^d}^2 \left(1+\beta_{21}Y_1(t)\right), \\ \mathrm{Var}_f(t) &= {\delta_1^f}^2 Y_1(t) + {\delta_3^f}^2 \left(1+\beta_{31}Y_1(t)\right), \\ \mathrm{Cov}_{df}(t) &= \delta_1^d \delta_1^f Y_1(t). \end{aligned}$$

It is clear that the correlation of $r_d(t)$ and $r_f(t)$ cannot switch sign over time.

$\underline{IA_{2;0,1,1}(3;1,1,1)}$

This model includes two square-root local factors, $Y_1(t)$ and $Y_2(t)$, and one Gaussian common factor, $Y_3(t)$. Interest rate of each local market is given as:

$$\begin{aligned} r_d(t) &= \delta_0^d + \delta_1^d Y_1(t) + \delta_3^d Y_3(t), \\ r_f(t) &= \delta_0^f + \delta_2^f Y_2(t) + \delta_3^f Y_3(t). \end{aligned}$$

The SDEs of the state variables are

$$\begin{aligned} d\begin{pmatrix} Y_1(t) \\ Y_2(t) \\ Y_3(t) \end{pmatrix} &= \begin{pmatrix} \kappa_{11} & 0 & 0 \\ 0 & \kappa_{22} & 0 \\ 0 & 0 & \kappa_{33} \end{pmatrix} \begin{pmatrix} \theta_1 & - & Y_1(t) \\ \theta_2 & - & Y_2(t) \\ & - & Y_3(t) \end{pmatrix} dt \\ &+ \begin{pmatrix} \sqrt{Y_1(t)} & 0 & 0 \\ 0 & \sqrt{Y_2(t)} & 0 \\ 0 & 0 & 1 \end{pmatrix} dW(t). \end{aligned}$$

Thus the instantaneous correlation of the interest rates is represented as:

$$\mathrm{Corr}_{df}(t) = \frac{\mathrm{Cov}_{df}(t)}{\sqrt{\mathrm{Var}_d(t)}\sqrt{\mathrm{Var}_f(t)}}, \tag{78}$$

where

$$
\begin{aligned}
\mathrm{Var}_d(t) &= \delta_1^{d\,2} Y_1(t) + \delta_3^{d\,2}, \\
\mathrm{Var}_f(t) &= \delta_2^{f\,2} Y_2(t) + \delta_3^{f\,2}, \\
\mathrm{Cov}_{df}(t) &= \delta_3^d \delta_3^f.
\end{aligned}
$$

Therefore $IA_{2;0,1,1}(3;1,1,1)$ is unable to generate the sign-switching correlation even though it includes two square-root factors.

Table 1: Classifications of $\mathbb{IA}_{m;m_c,m_d,m_f}(N;N_c,N_d,N_f)$ with $N=3$ and $N=4$

$N=3$*: Common factor models*

	$IA_{0;0,0,0}(3;3,0,0)$	$\mathbb{IA}_{1;1,0,0}(3;3,0,0)$	$\mathbb{IA}_{2;2,0,0}(3;3,0,0)$	$IA_{3;3,0,0}(3;3,0,0)$
Y^{B_c}		Y_1	Y_1, Y_2	Y_1, Y_2, Y_3
Y^{B_d}				
Y^{B_f}				
Y^{D_c}	Y_1, Y_2, Y_3	Y_2, Y_3	Y_3	
Y^{D_d}				
Y^{D_f}				

$N=3$*: Common and local factor models*

	$IA_{0;0,0,0}(3;1,1,1)$	$IA_{1;1,0,0}(3;1,1,1)$	$IA_{2;0,1,1}(3;1,1,1)$	$IA_{3;1,1,1}(3;1,1,1)$
Y^{B_c}		Y_1		Y_1
Y^{B_d}			Y_1	Y_2
Y^{B_f}			Y_2	Y_3
Y^{D_c}	Y_1		Y_3	
Y^{D_d}	Y_2	Y_2		
Y^{D_f}	Y_3	Y_3		

$N=4$*: Common and local factor models*

	$IA_{0;0,0,0}(4;2,1,1)$	$IA_{1;1,0,0}(4;2,1,1)$	$IA_{2;2,0,0}(4;2,1,1)$
Y^{B_c}		Y_1	Y_1, Y_2
Y^{B_d}			
Y^{B_f}			
Y^{D_c}	Y_1, Y_2	Y_2	
Y^{D_d}	Y_3	Y_3	Y_3
Y^{D_f}	Y_4	Y_4	Y_4

	$IA_{2;0,1,1}(4;2,1,1)$	$IA_{3;1,1,1}(4;2,1,1)$	$IA_{4;2,1,1}(4;2,1,1)$
Y^{B_c}		Y_1	Y_1, Y_2
Y^{B_d}	Y_1	Y_2	Y_3
Y^{B_f}	Y_2	Y_3	Y_4
Y^{D_c}	Y_3, Y_4	Y_4	
Y^{D_d}			
Y^{D_f}			

The table presents classifications for three-and four-factor $\mathbb{IATSM}$s, wherein the factor structures of the two local markets are symmetric.

Table 2: Specification Tests of Two-Country Term Structure Models

	$\mathbb{IA}_{1;1,0,0}(3;3,0,0)$		$\mathbb{IA}_{2;2,0,0}(3;3,0,0)$		$\mathbb{IA}_{2;2,0,0}(4;2,1,1)$		$\mathbb{IA}_{3;1,1,1}(4;2,1,1)$		$\mathbb{IQ}(3;1,1,1)$
δ_0^d	0.0005 (0.0039)	δ_0^d	-0.0266 (0.0007)	δ_0^d	-0.0054 (0.0026)	δ_0^d	0.0031 (0.0009)	α^d	0.0151 (0.0015)
δ_0^f	0.0719 (0.0051)	δ_0^f	-0.0287 (0.0040)	δ_0^f	-0.0132 (0.0021)	δ_0^f	0.0070 (0.0027)	α^f	0.0094 (0.0004)
δ_1^d	0.0091 (0.0005)	δ_1^d	0.0222 (0.0014)	δ_1^d	0.0133 (0.0007)	δ_1^d	0.0234 (0.0008)	β_1^f	-0.2464 (0.0467)
δ_2^d	0.0096 (0.0011)	δ_2^d	0.0116 (0.0006)	δ_2^d	0.0148 (0.0005)	δ_2^d	0.0084 (0.0006)	Ψ_{12}^d	0.7290 (0.0995)
δ_3^d	0.0158 (0.0007)	δ_3^d	0.0034 (0.0003)	δ_3^d	0.0069 (0.0007)	δ_4^d	0.0627 (0.0329)	Ψ_{11}^f	16.2748 (1.8739)
δ_1^f	-0.0077 (0.0006)	δ_1^f	-0.0017 (0.0019)	δ_1^f	0.0063 (0.0011)	δ_1^f	-0.0014 (0.0022)	Ψ_{13}^f	-3.8269 (0.1344)
δ_2^f	0.0163 (0.0017)	δ_2^f	0.0162 (0.0011)	δ_2^f	0.0124 (0.0005)	δ_3^f	0.0072 (0.0008)	Φ_{01}	0.0869 (0.0002)
δ_3^f	0.0060 (0.0008)	δ_3^f	-0.0015 (0.0002)	δ_4^f	0.0000 (0.0000)	δ_4^f	0.0010 (0.0002)	Φ_{02}	0.0007 (0.0039)
κ_{11}	0.6417 (0.1113)	κ_{11}	4.0466 (0.4746)	κ_{11}	5.0826 (0.4310)	κ_{11}	1.3692 (0.2159)	Φ_{03}	0.0004 (0.0044)
κ_{21}	-0.1445 (0.0711)	κ_{21}	-5.3938 (0.9310)	κ_{21}	-4.5238 (0.4929)	κ_{21}	-1.2533 (0.7259)	Φ_{111}	-3.5503 (0.0140)
κ_{31}	4.2738 (0.2444)	κ_{31}	0.5633 (0.8155)	κ_{31}	-3.4945 (0.6799)	κ_{31}	-2.3433 (0.6305)	Φ_{121}	-8.5398 (0.1561)
κ_{22}	1.1839 (0.1727)	κ_{12}	-1.4757 (0.1482)	κ_{41}	-434.8295(704.60)	κ_{41}	-9.7932 (2.1248)	Φ_{131}	8.9264 (0.4025)
κ_{32}	-6.1101 (0.2940)	κ_{22}	4.7562 (0.3439)	κ_{12}	-2.3140 (0.3484)	κ_{22}	1.2744 (0.5346)	Φ_{122}	-1.0051 (0.0231)
κ_{23}	1.2698 (0.0958)	κ_{32}	1.6760 (0.6785)	κ_{22}	4.3772 (0.4575)	κ_{33}	1.5320 (0.1971)	Φ_{133}	-1.3153 (0.0093)
κ_{33}	2.0627 (0.1053)	κ_{33}	1.0166 (0.1799)	κ_{32}	4.1115 (0.4559)	κ_{44}	0.4467 (0.2351)	Σ_{11}	0.0311 (0.0002)
θ_1	6.2545 (0.2228)	θ_1	1.6105 (0.1226)	κ_{42}	287.1631 (32.312)	θ_1	1.2968 (0.0595)	Σ_{22}	0.0215 (0.0009)
β_{21}	0.0576 (0.0291)	θ_2	3.2597 (0.1896)	κ_{33}	1.4991 (0.0770)	θ_2	1.7367 (0.1648)	Σ_{33}	0.1216 (0.0018)
β_{31}	0.5138 (0.1403)	β_{31}	4.2887 (0.6485)	κ_{44}	8.2235 (15.449)	θ_3	2.4508 (0.5453)	η_{01}^d	0.0084 (0.0003)
λ_1^d	0.0273 (0.0008)	β_{32}	1.3803 (0.2030)	θ_1	1.6545 (0.0746)	β_{41}	3.2434 (0.2723)	η_{02}^d	0.0853 (0.0007)
λ_2^d	-0.0203 (0.0051)	λ_1^d	-0.1037 (0.0033)	θ_2	2.2245 (0.1302)	λ_1^d	-0.0694 (0.0026)	η_{03}^d	-0.0080 (0.0006)
λ_3^d	-0.0661 (0.0050)	λ_2^d	0.0454 (0.0022)	β_{31}	0.0788 (0.2862)	λ_2^d	0.0593 (0.0041)	η_{111}^d	2.4721 (0.0429)
		λ_3^d	0.0150 (0.0008)	β_{32}	7.9772 (1.2889)	λ_3^d	-0.0006 (0.0017)	η_{121}^d	1.2881 (0.0703)
				β_{41}	3.8303 (0.8015)	λ_4^d	0.0521 (0.0015)	η_{131}^d	-4.1663 (0.1277)
				β_{42}	2.1473 (0.4556)			η_{122}^d	-0.2150 (0.0027)
				λ_1^d	-0.0825 (0.0016)			η_{133}^d	0.5633 (0.0050)
				λ_2^d	0.0447 (0.0022)				
				λ_3^d	0.0057 (0.0004)				
				λ_4^d	0.0356 (0.0029)				
χ^2	54.564		46.657		32.451		60.601		23.799
df	18		17		11		16		14
z	6.094		5.086		4.573		7.884		1.852

The table presents parameter estimates and goodness-of-fit tests for five two-country term structure models. The model and parameters are described in Section 3 and Section 4. The columns present parameter estimates for $\mathbb{IA}_{1;1,0,0}(3;3,0,0)$, $\mathbb{IA}_{2;2,0,0}(3;3,0,0)$, $\mathbb{IA}_{2;2,0,0}(4;2,1,1)$, $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$, and $\mathbb{IQ}(3;1,1,1)$. Standard errors are given in parentheses. The table also presents χ^2 statistics for the goodness-of-fit of the models, degrees of freedom, and corresponding z-statistic that adjusts for degrees of freedom across the models and is distributed $N(0,1)$.

Table 3: EMM Diagnostics of Two-Country Term Structure Models

Coefficient	$\mathbb{A}_{1;1,0,0}(3;3,0,0)$	$\mathbb{A}_{2;2,0,0}(3;3,0,0)$	$\mathbb{A}_{2;2,0,0}(4;2,1,1)$	$\mathbb{A}_{3;1,1,1}(4;2,1,1)$	$\mathbb{Q}(3;1,1,1)$
$a_0(1)$	1.324	-0.447	-0.283	-1.246	1.342
$a_0(2)$	-1.491	-2.264	-1.681	-1.923	-0.542
$a_0(3)$	-3.818	-2.997	-2.461	-2.391	-2.361
$a_0(4)$	-1.106	-0.618	-1.394	-1.345	-1.912
$a_0(5)$	0.845	0.568	-0.115	-0.032	0.130
$a_o(6)$	-0.570	-0.969	-1.123	-0.892	0.238
$a_0(7)$	0.517	1.300	1.107	2.313	1.100
$a_0(8)$	-0.530	-1.583	-1.474	-0.587	-0.231
$a_0(9)$	0.410	0.561	0.745	0.655	0.834
$a_0(10)$	3.202	3.201	2.875	3.440	0.741
$a_0(11)$	-0.864	1.082	1.337	0.447	0.667
$a_0(12)$	-1.874	-1.912	-0.260	-2.752	-0.809
$b_0(1)$	0.993	-0.091	-0.357	-0.140	0.791
$b_0(2)$	0.166	0.029	-0.046	-0.280	-0.996
$b_0(3)$	0.927	0.382	0.151	-0.291	-0.391
$B(1,1)$	-0.788	-0.275	-0.433	-0.092	-0.954
$B(2,1)$	0.782	0.327	0.663	0.113	1.029
$B(3,1)$	0.692	-0.155	0.573	-0.127	1.064
$B(1,2)$	-0.055	0.439	0.730	0.852	0.826
$B(2,2)$	-0.268	-0.462	-0.830	-0.824	-0.268
$B(3,2)$	-0.146	0.204	-0.599	-0.305	-0.006
$B(1,3)$	0.115	-0.090	-0.408	0.211	0.218
$B(2,3)$	-0.116	-0.184	0.282	-0.946	-1.346
$B(3,3)$	0.096	-0.151	0.437	-0.652	-1.088
$R_0(1,1)$	1.386	2.080	2.488	2.549	0.069
$R_0(1,2)$	-1.326	0.372	0.470	-0.485	0.702
$R_0(2,2)$	1.687	0.127	0.221	0.870	-0.348
$R_0(1,3)$	1.650	1.666	2.106	2.441	-0.131
$R_0(2,3)$	0.952	-0.501	-0.037	0.431	-0.160
$R_0(3,3)$	-0.932	-0.436	-0.259	-0.707	0.274
$P_1(1,1)$	2.777	2.162	1.824	1.902	-0.229
$P_1(2,2)$	-2.023	0.110	0.561	0.565	1.063
$P_1(3,3)$	0.693	-1.001	-0.996	-0.813	-0.229
$Q_1(1,1)$	3.295	2.672	2.261	2.778	-0.103
$Q_1(2,2)$	-2.201	-0.269	-0.157	-0.735	0.308
$Q_1(3,3)$	1.422	-0.272	-0.391	0.415	0.131
$W_1(1,1)$	0.113	0.874	-0.011	0.246	-0.405
$W_1(2,2)$	0.090	0.854	1.038	1.195	0.681
$W_1(3,3)$	-0.189	-0.726	-0.753	-0.445	0.222

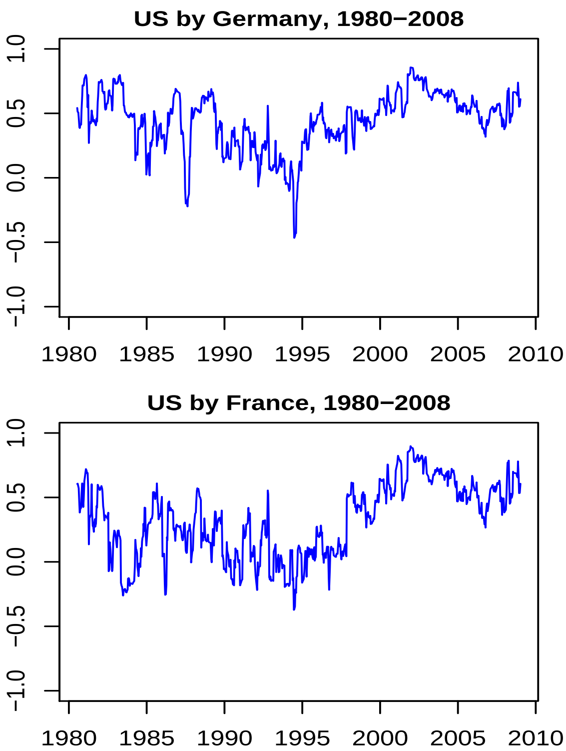
The coefficients labeled $a_0(1)$-$a_0(12)$ denote the coefficients of the Hermite polynomial of the SNP model, $b_0(1)$-$B(3,3)$ denotes the VAR terms of the SNP model, and $R_0(1,1)$-$W_1(3,3)$ denote the modified BEKK-GARCH terms of the SNP model. The table presents t-statistics for the test of the null hypothesis that the score with respect to the coefficient is equal to 0.

Table 4: Conditional Correlations and Two-Country Business Cycles

Parameter	α	β_{EE}	β_{RE}	β_{ER}
Estimate	0.4526	-0.3220	-0.0996	-0.2531
Standard error	0.0250	0.0305	0.0454	0.0323
pvalue	0.0000	0.0000	0.0140	0.0000
Hypothesis			z-value	p-value
H_0: $\beta_{RE} - \beta_{ER} = 0$			3.5642	0.0002
H_0: $\beta_{ER} - \beta_{EE} = 0$			2.5634	0.0052

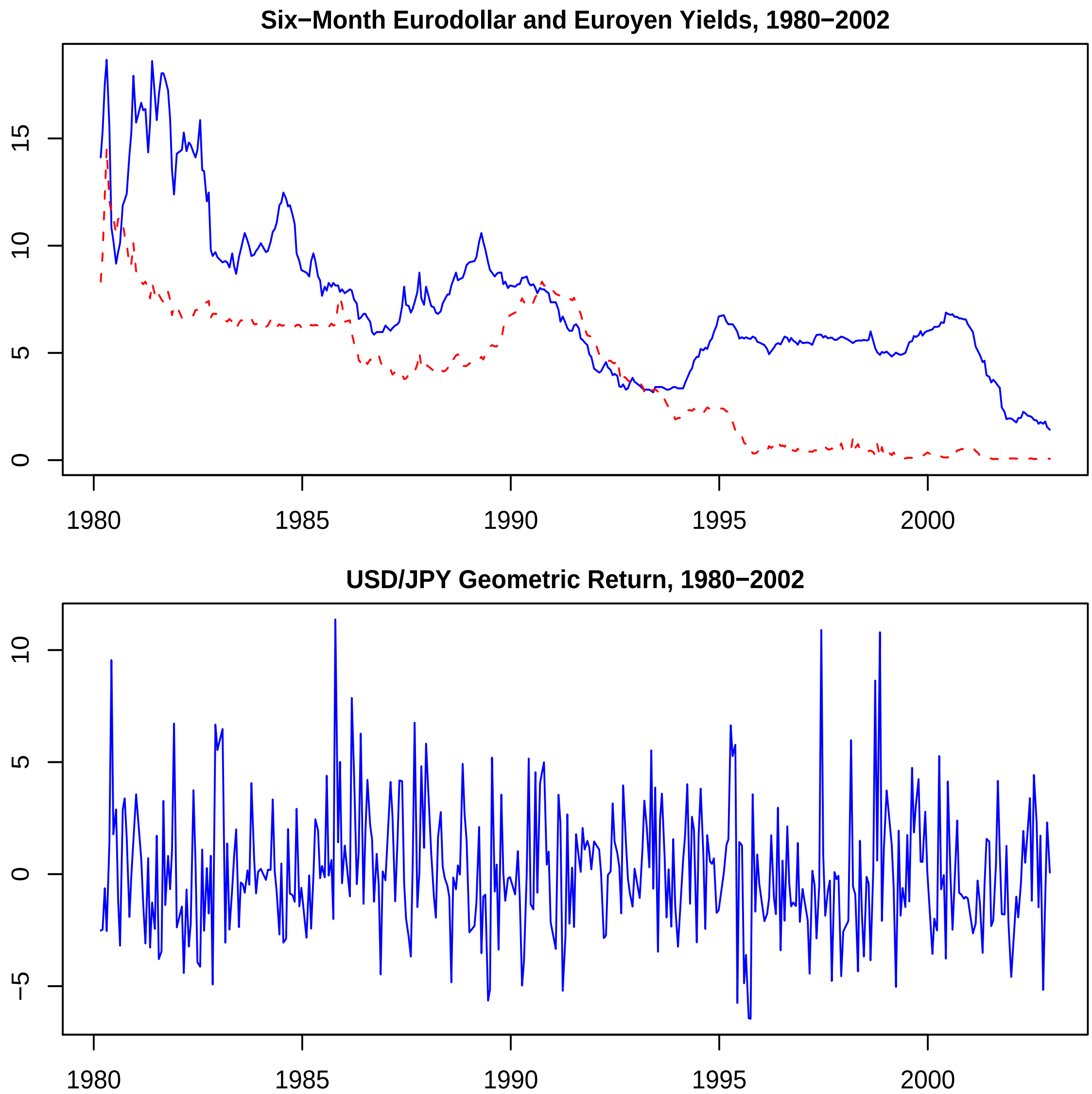
The upper part of the table summarizes the GMM-based regression results where the dependent variable is conditional correlation and explanatory variables are indicator dummies for different regimes in two-country business cycles. We report the regression coefficients, standard errors and corresponding p-values. In the bottom part of the table, we document the Wald test results for testing whether mean correlations are identical across different regimes in two-country business cycles. Standard errors are serial correlation and heteroskedasticity-adjusted using Newey and West (1987).

Figure 1: **Conditional Correlations among International Interest Rates**



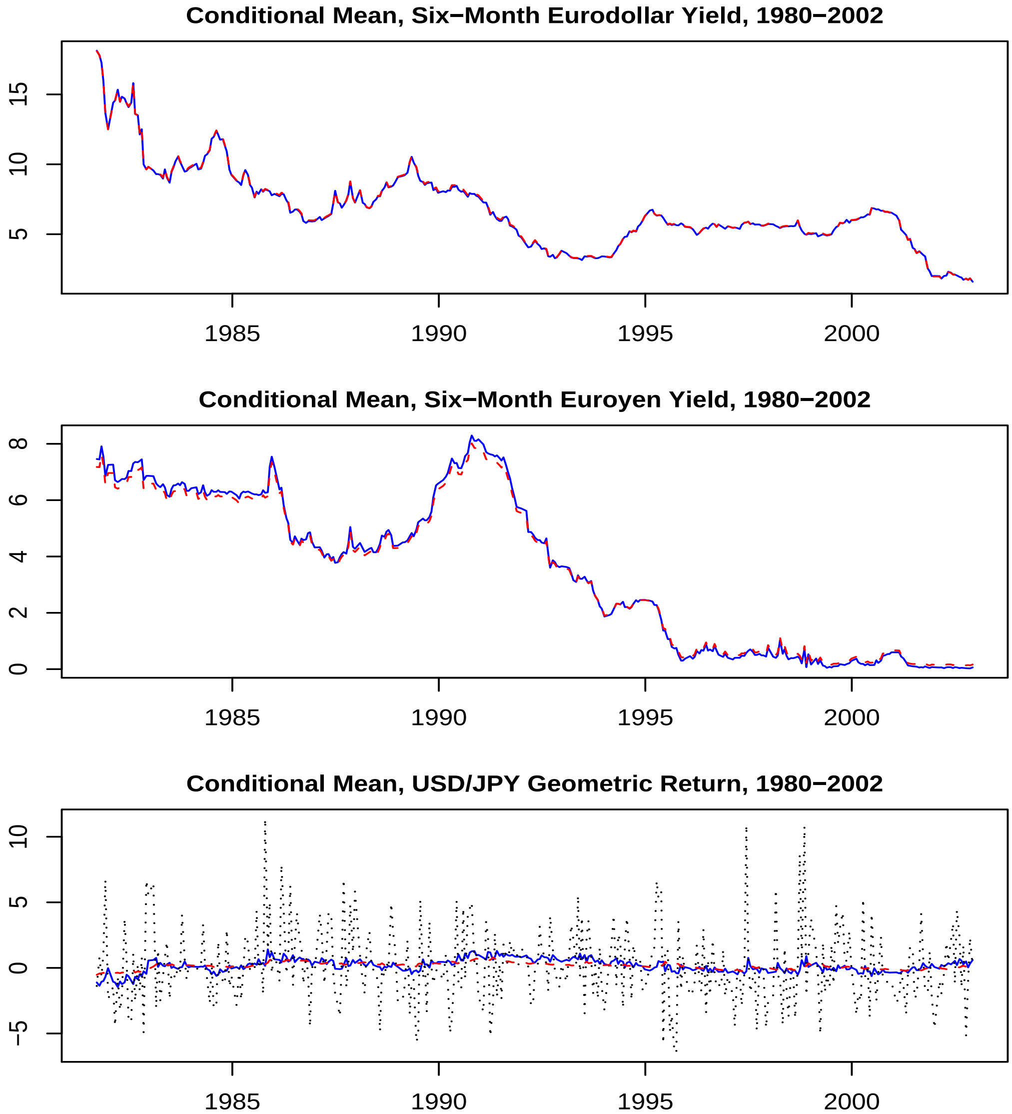
The plots present the conditional correlations among U.S. and two developed countries, and among U.S. and two emerging countries. One-year Eurocurrency interest rates are used for the country pairs of U.S. and each developed country. Two-year government bond interest rates are exploited for the pairs of U.S. and each emerging country. A bivariate BEKK-GARCH model is estimated for the bi-weekly changes of the interest rate for each country pair.

Figure 2: **Six-Month Eurodollar and Euroyen Yields, and USD/JPY Geometric Return**



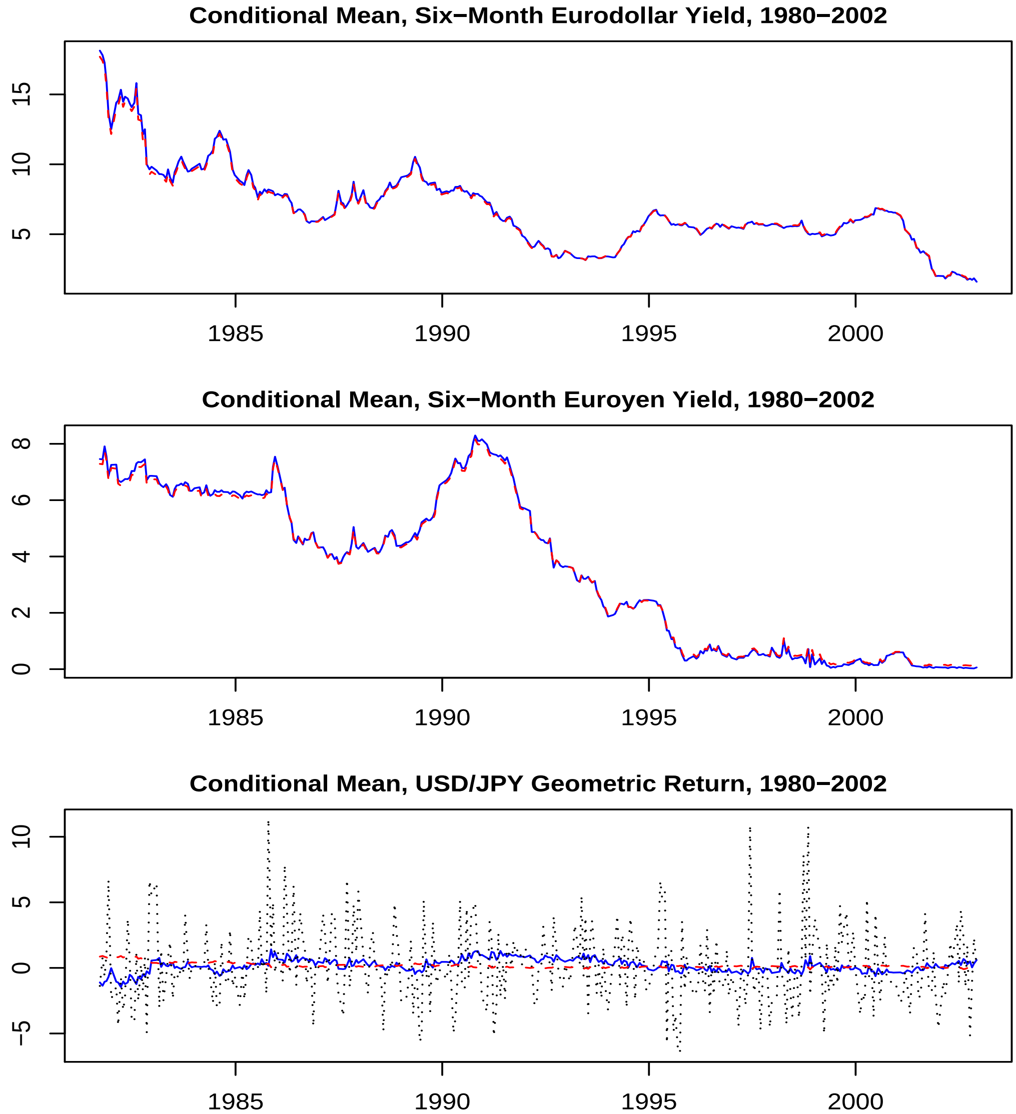
The plots present the six-month Eurodollar and Euroyen zero-coupon yields, and the tri-weekly U.S. Dollar per Japanese Yen geometric return data over the period January 29, 1980 through November 26, 2002. The Eurocurrency interest rates are taken from the Financial Times and the exchange rate data is obtained from Morgan Stanley Capital International (MSCI), both provided by Datastream. Tuesday-to-Tuesday data are sampled at a tri-weekly frequency.

Figure 3: **Reprojected Conditional Mean:** $\mathbb{I}\mathbb{A}_{1;1,0,0}(3;3,0,0)$



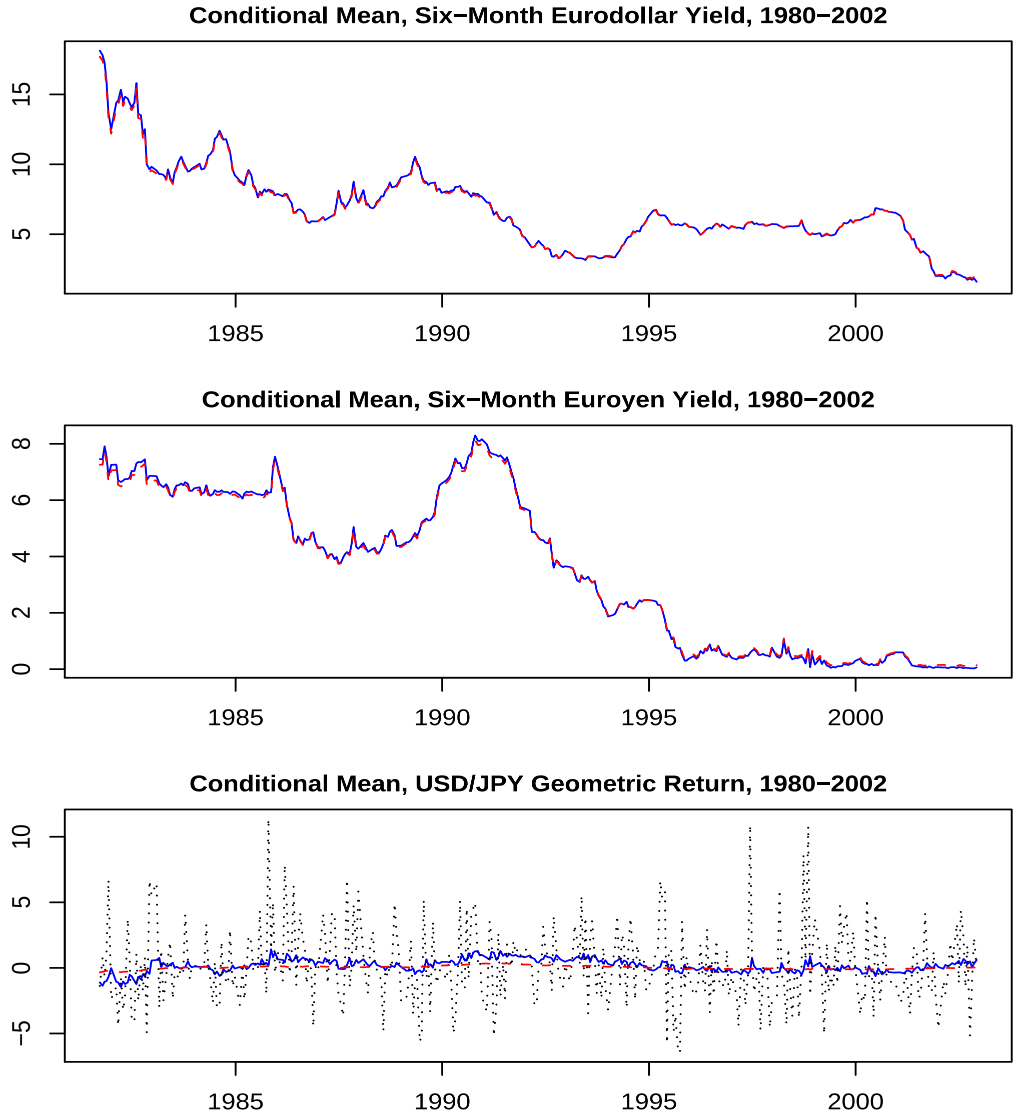
The plots present the reprojected conditional mean for the $\mathbb{I}\mathbb{A}_{1;1,0,0}(3;3,0,0)$ model against the projected conditional mean. The reprojected data are represented by dashed lines, whereas the projected data are represented by solid line. In the last plot, dotted line represents the actual USD/JPY geometric return.

Figure 4: **Reprojected Conditional Mean:** $\mathbb{IA}_{2;2,0,0}(3;3,0,0)$



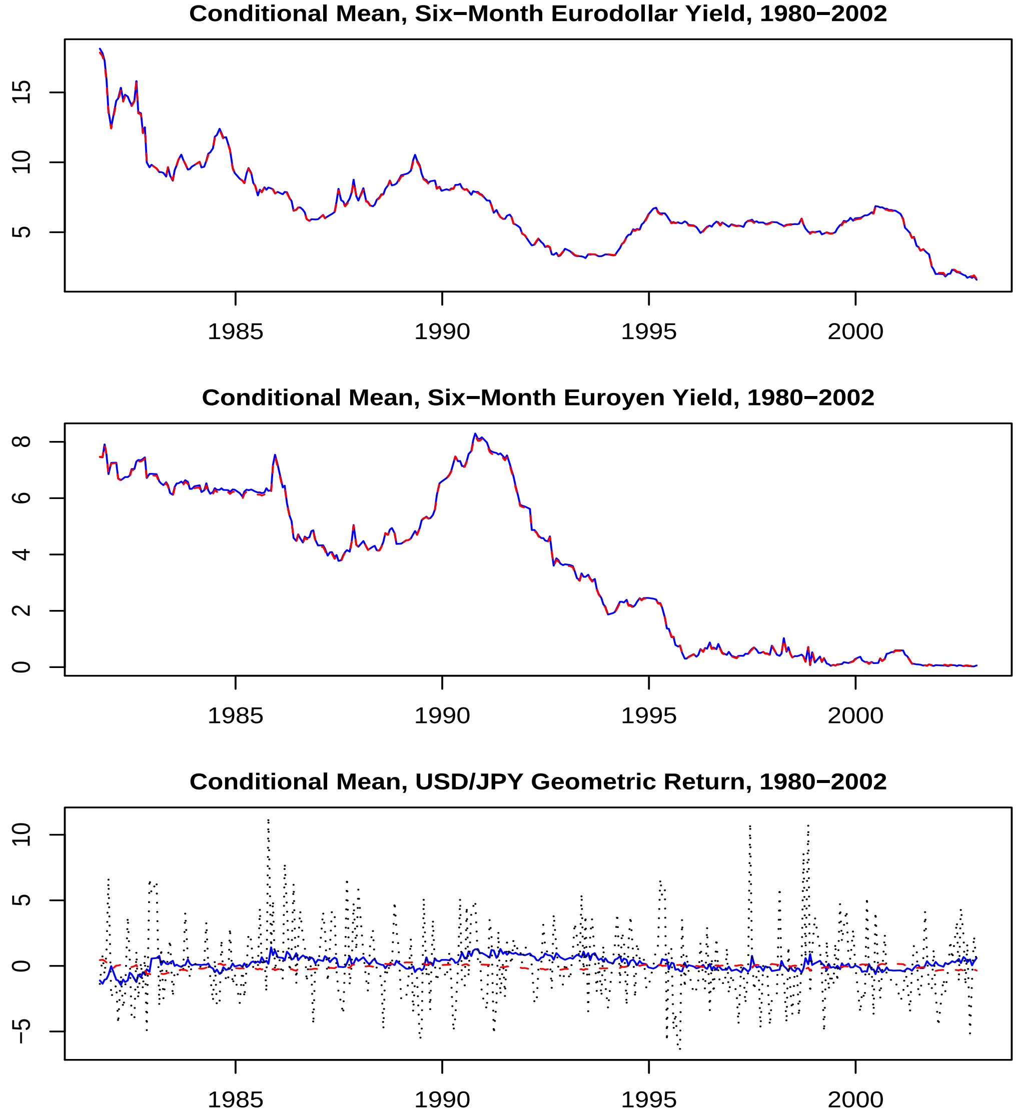
The plots present the reprojected conditional mean for the $\mathbb{IA}_{2;2,0,0}(3;3,0,0)$ model against the projected conditional mean. The reprojected data are represented by dashed lines, whereas the projected data are represented by solid line. In the last plot, dotted line represents the actual USD/JPY geometric return.

Figure 5: **Reprojected Conditional Mean:** $\mathbb{IA}_{2;2,0,0}(4;2,1,1)$



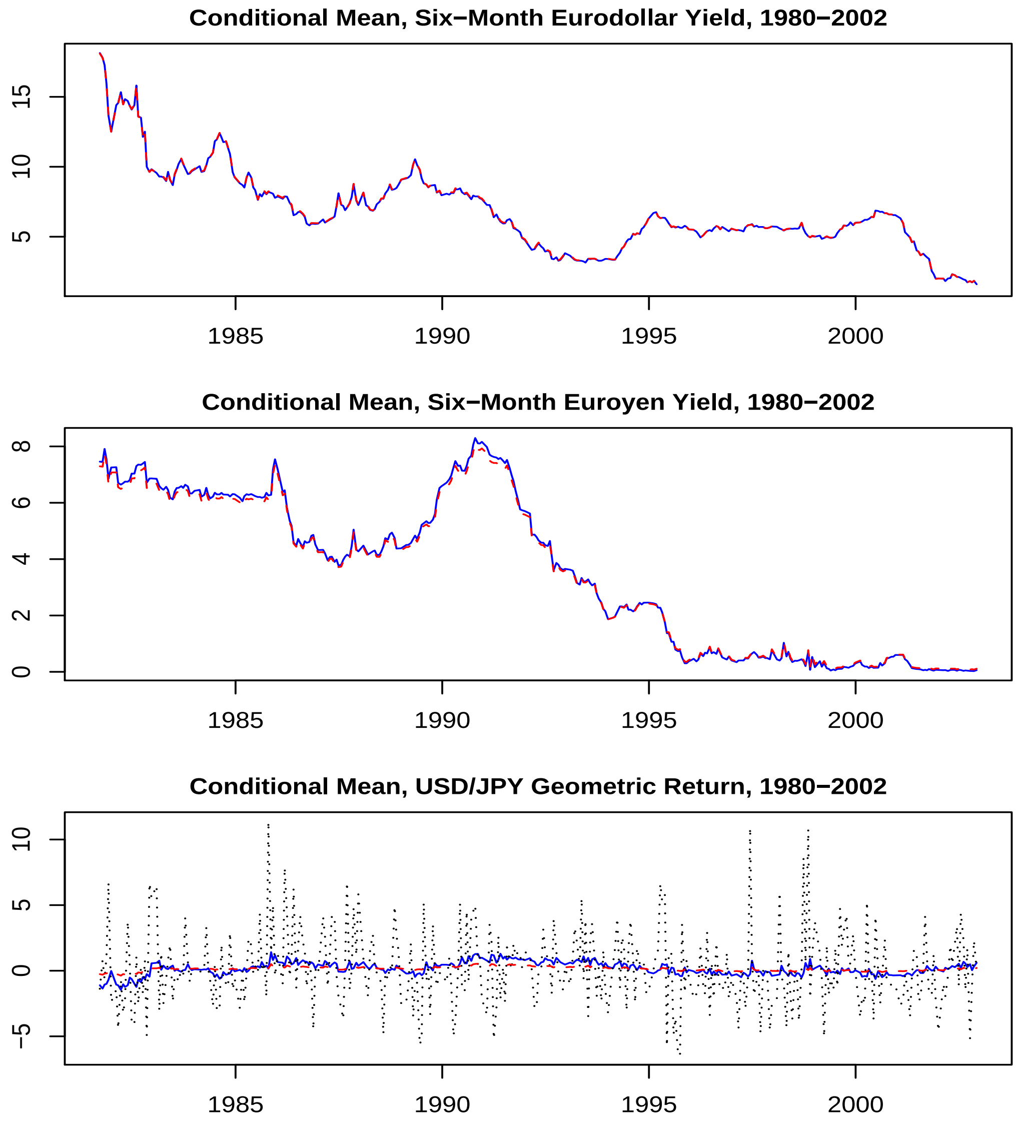
The plots present the reprojected conditional mean for the $\mathbb{IA}_{2;2,0,0}(4;2,1,1)$ model against the projected conditional mean. The reprojected data are represented by dashed lines, whereas the projected data are represented by solid line. In the last plot, dotted line represents the actual USD/JPY geometric return.

Figure 6: **Reprojected Conditional Mean:** $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$



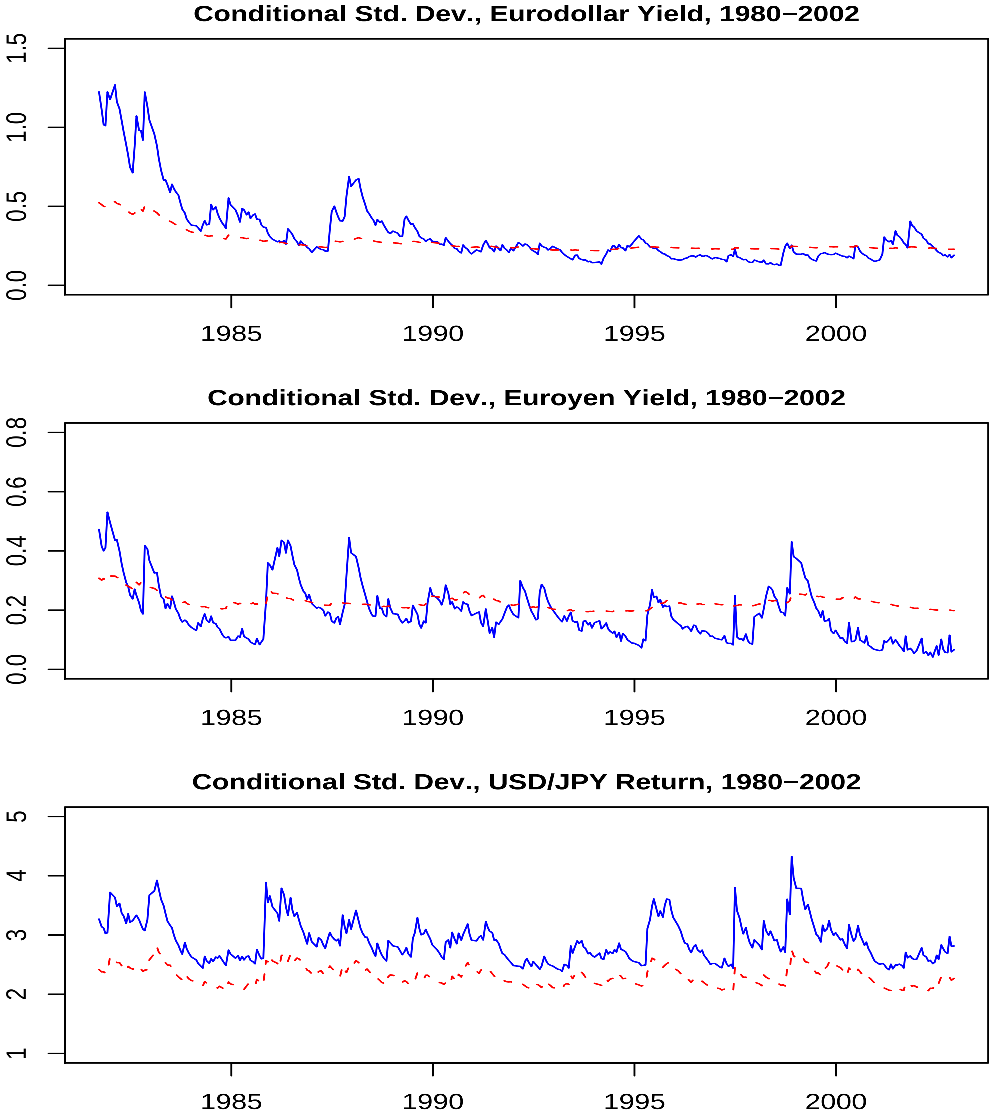
The plots present the reprojected conditional mean for the $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$ model against the projected conditional mean. The reprojected data are represented by dashed lines, whereas the projected data are represented by solid line. In the last plot, dotted line represents the actual USD/JPY geometric return.

Figure 7: **Reprojected Conditional Mean:** $\mathbb{IQ}(3; 1, 1, 1)$



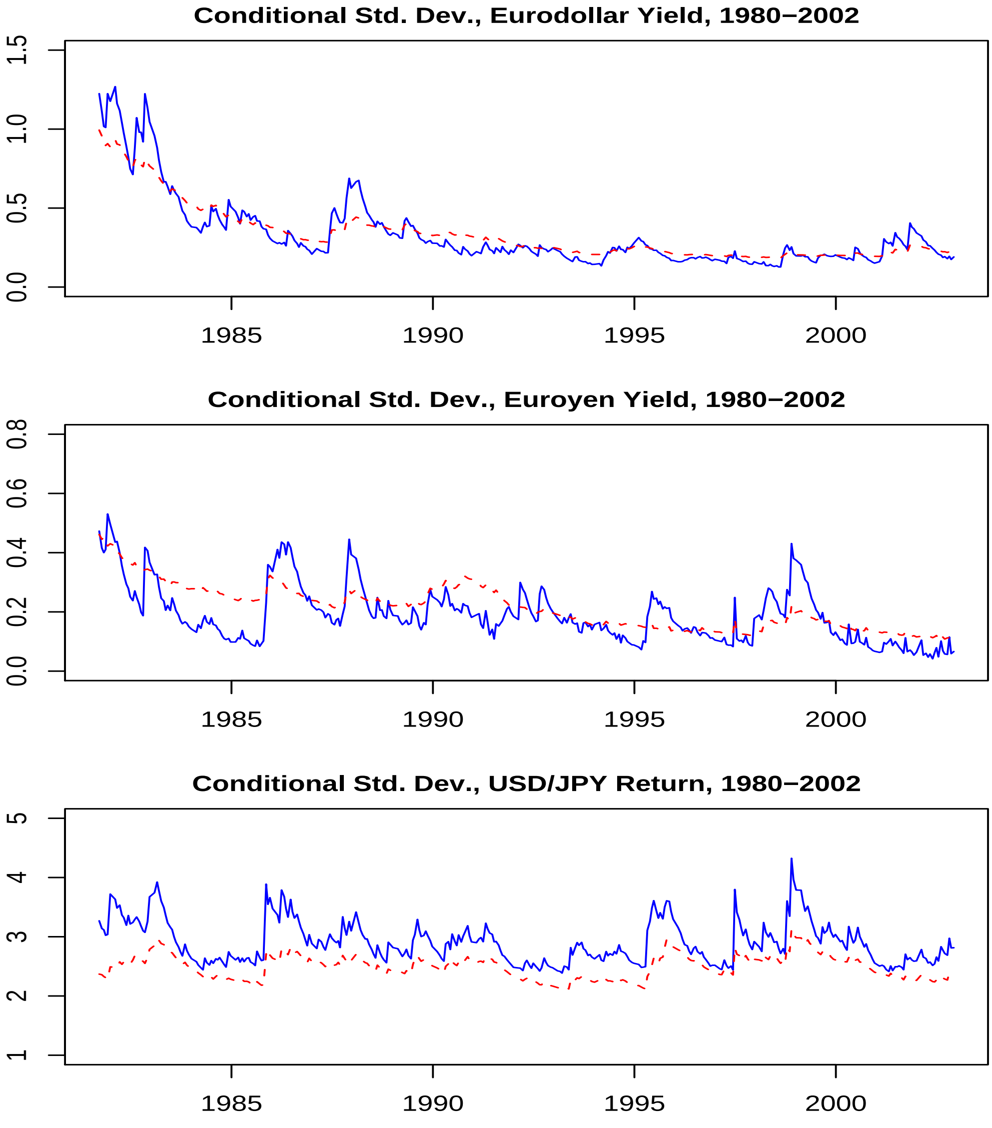
The plots present the reprojected conditional mean for the $\mathbb{IQ}(3; 1, 1, 1)$ model against the projected conditional mean. The reprojected data are represented by dashed lines, whereas the projected data are represented by solid line. In the last plot, dotted line represents the actual USD/JPY geometric return.

Figure 8: **Reprojected Conditional Volatility:** $\mathbb{IA}_{1;1,0,0}(3;3,0,0)$



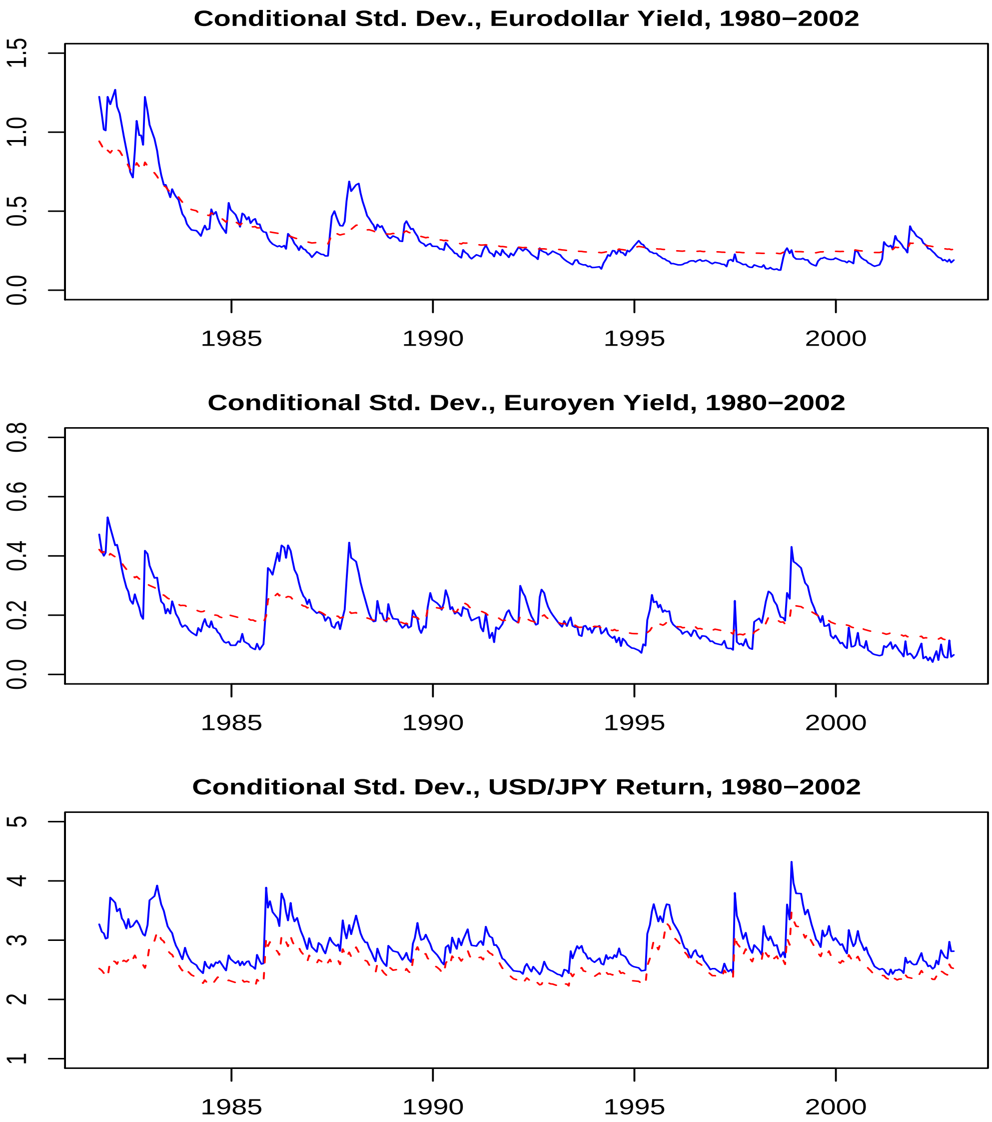
The plots present the reprojected conditional volatility for the $\mathbb{IA}_{1;1,0,0}(3;3,0,0)$ model against the projected conditional volatility. The reprojected data are represented by dashed lines, whereas the projected data are represented by solid line.

Figure 9: **Reprojected Conditional Volatility:** $\mathbb{IA}_{2;2,0,0}(3;3,0,0)$



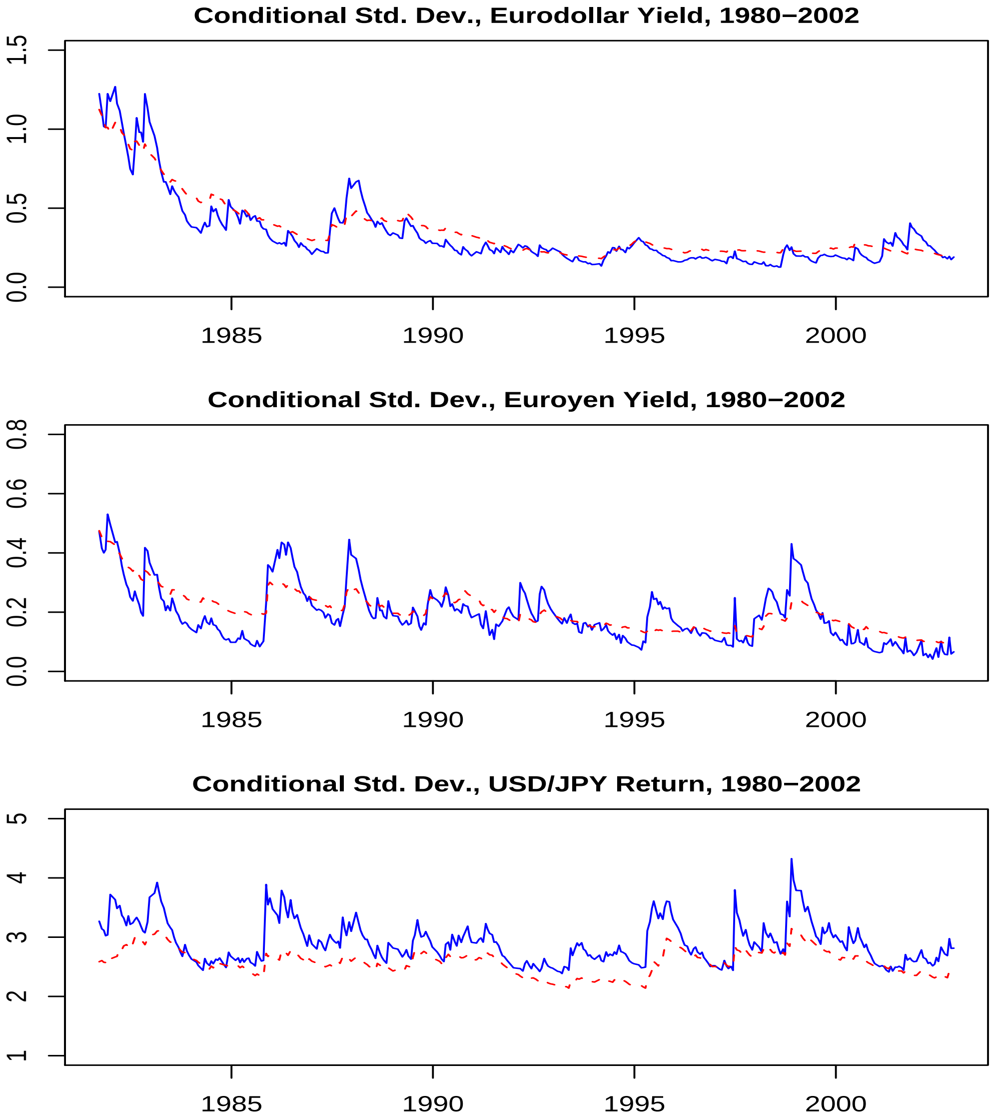
The plots present the reprojected conditional volatility for the $\mathbb{IA}_{2;2,0,0}(3;3,0,0)$ model against the projected conditional volatility. The reprojected data are represented by dashed lines, whereas the projected data are represented by solid line.

Figure 10: **Reprojected Conditional Volatility:** $\mathbb{IA}_{2;2,0,0}(4;2,1,1)$



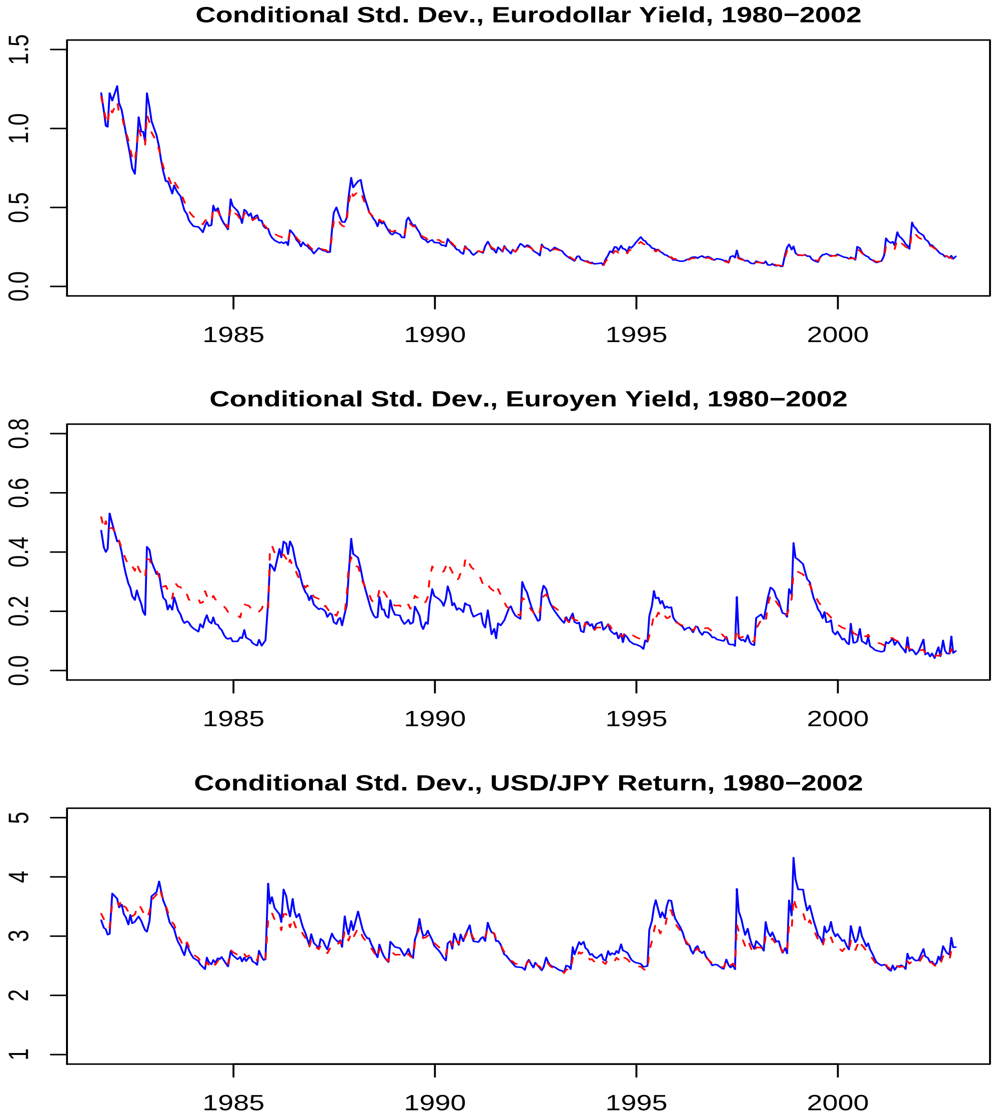
The plots present the reprojected conditional volatility for the $\mathbb{IA}_{2;2,0,0}(4;2,1,1)$ model against the projected conditional volatility. The reprojected data are represented by dashed lines, whereas the projected data are represented by solid line.

Figure 11: **Reprojected Conditional Volatility:** $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$



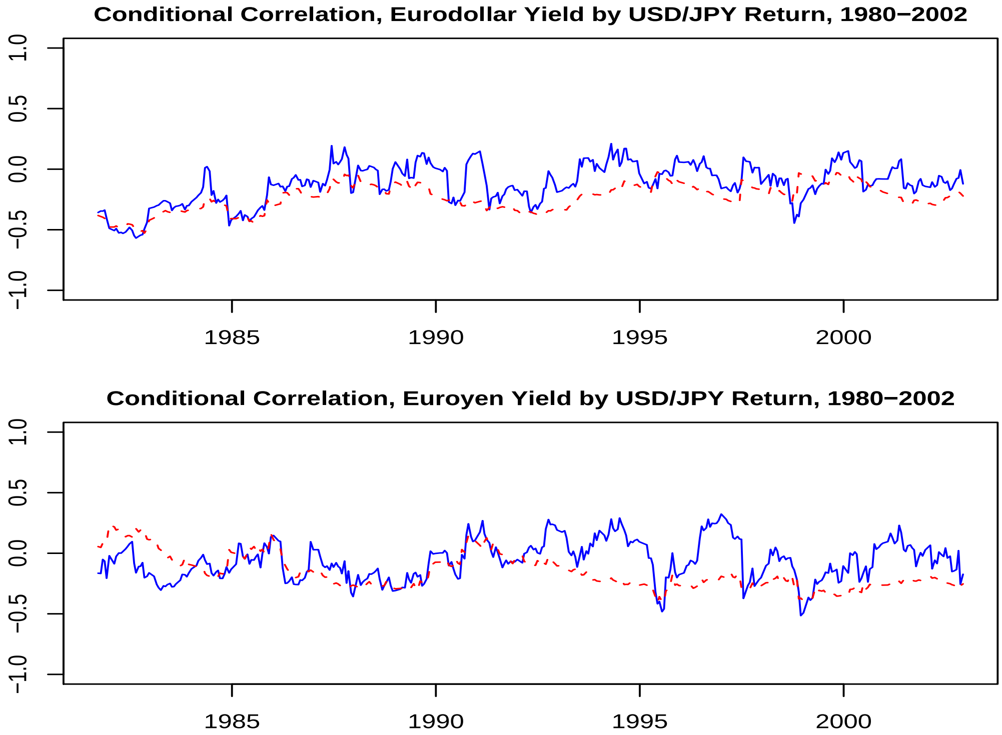
The plots present the reprojected conditional volatility for the $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$ model against the projected conditional volatility. The reprojected data are represented by dashed lines, whereas the projected data are represented by solid line.

Figure 12: **Reprojected Conditional Volatility:** $\mathbb{IQ}(3;1,1,1)$



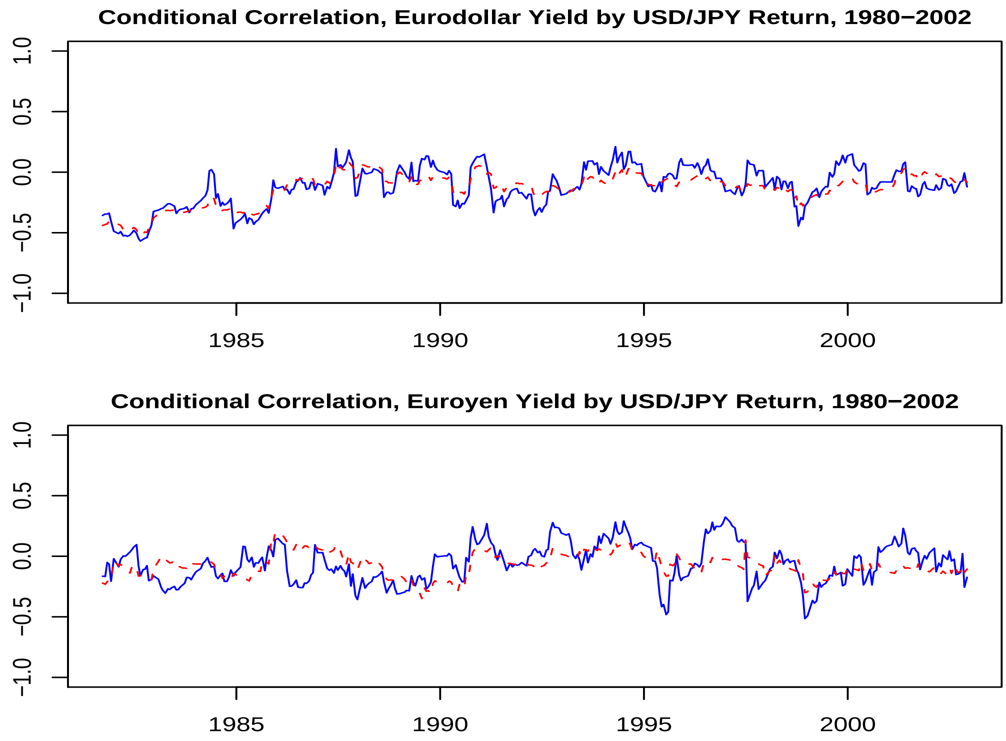
The plots present the reprojected conditional volatility for the $\mathbb{IQ}(3;1,1,1)$ model against the projected conditional volatility. The reprojected data are represented by dashed lines, whereas the projected data are represented by solid line.

Figure 13: **Reprojected Conditional Correlation:** $\mathbb{IA}_{1;1,0,0}(3;3,0,0)$



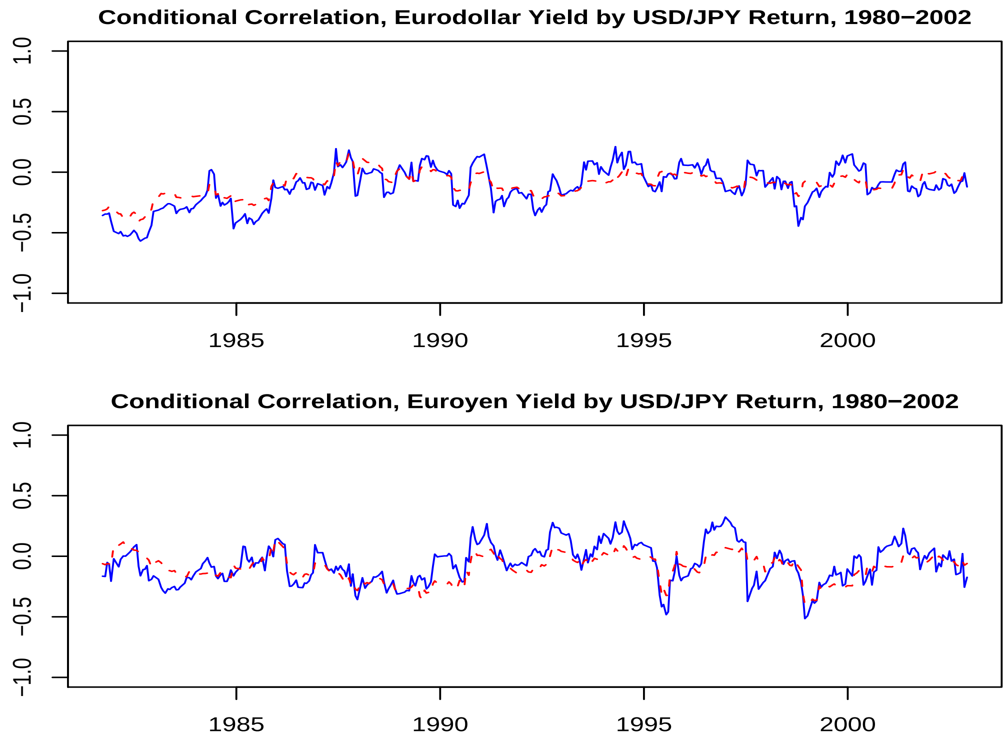
The plots present the reprojected conditional correlation for the $\mathbb{IA}_{1;1,0,0}(3;3,0,0)$ model against the projected conditional correlation. The reprojected data are represented by dashed lines, whereas the projected data are represented by solid line.

Figure 14: **Reprojected Conditional Correlation:** $\mathbb{IA}_{2;2,0,0}(3;3,0,0)$



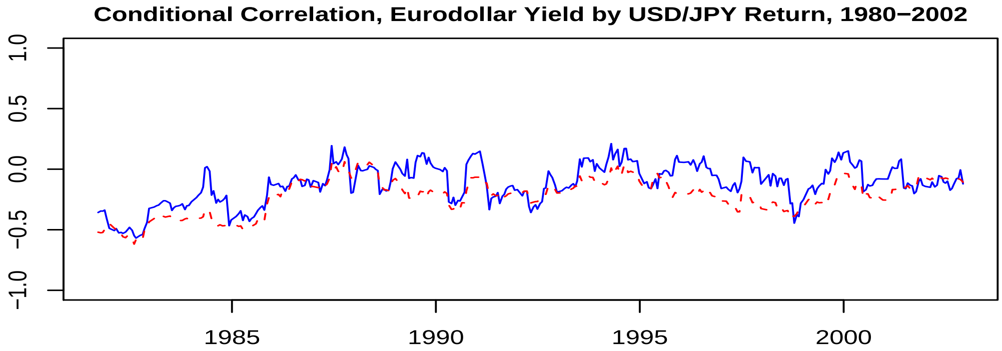
The plots present the reprojected conditional correlation for the $\mathbb{IA}_{2;2,0,0}(3;3,0,0)$ model against the projected conditional correlation. The reprojected data are represented by dashed lines, whereas the projected data are represented by solid line.

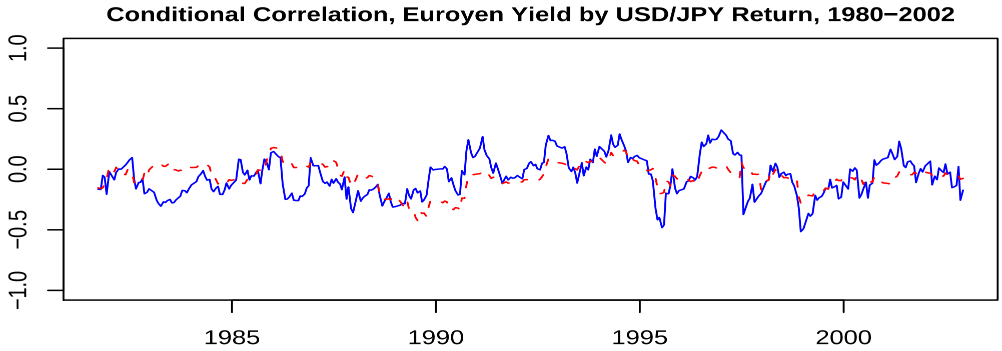
Figure 15: **Reprojected Conditional Correlation:** $\mathbb{IA}_{2;2,0,0}(4;2,1,1)$



The plots present the reprojected conditional correlation for the $\mathbb{IA}_{2;2,0,0}(4;2,1,1)$ model against the projected conditional correlation. The reprojected data are represented by dashed lines, whereas the projected data are represented by solid line.

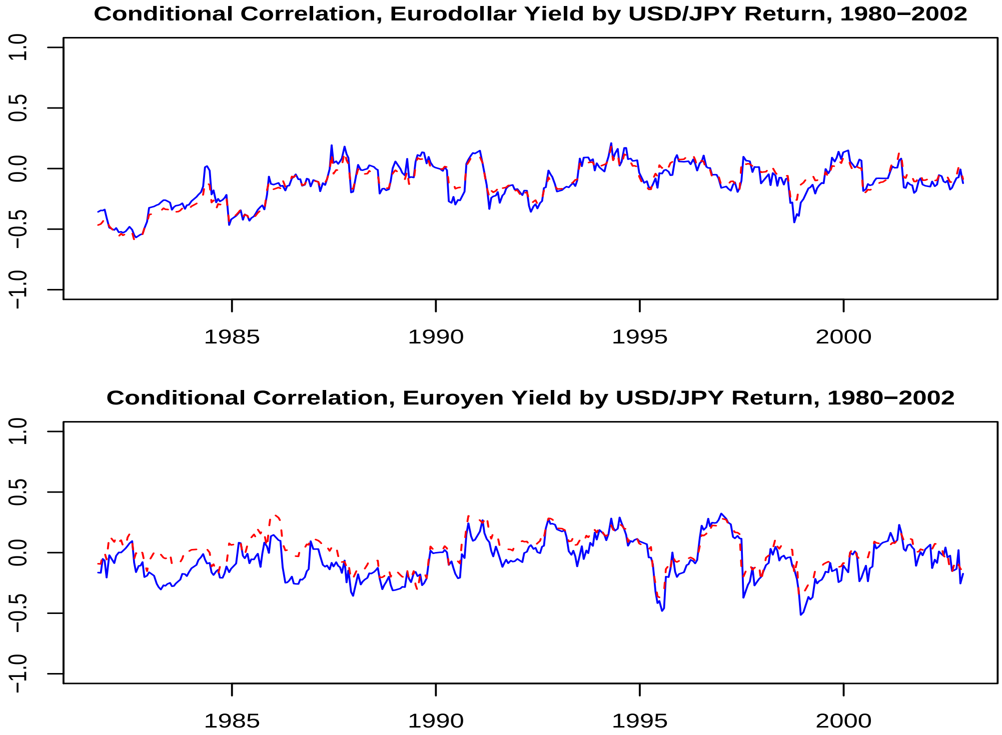
Figure 16: **Reprojected Conditional Correlation:** $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$





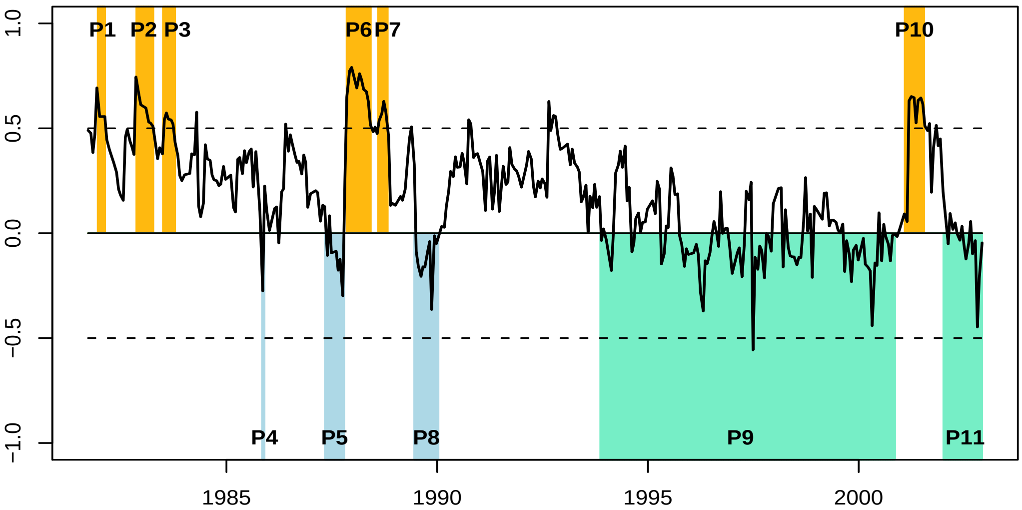
The plots present the reprojected conditional correlation for the $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$ model against the projected conditional correlation. The reprojected data are represented by dashed lines, whereas the projected data are represented by solid line.

Figure 17: **Reprojected Conditional Correlation:** $\mathbb{IQ}(3;1,1,1)$



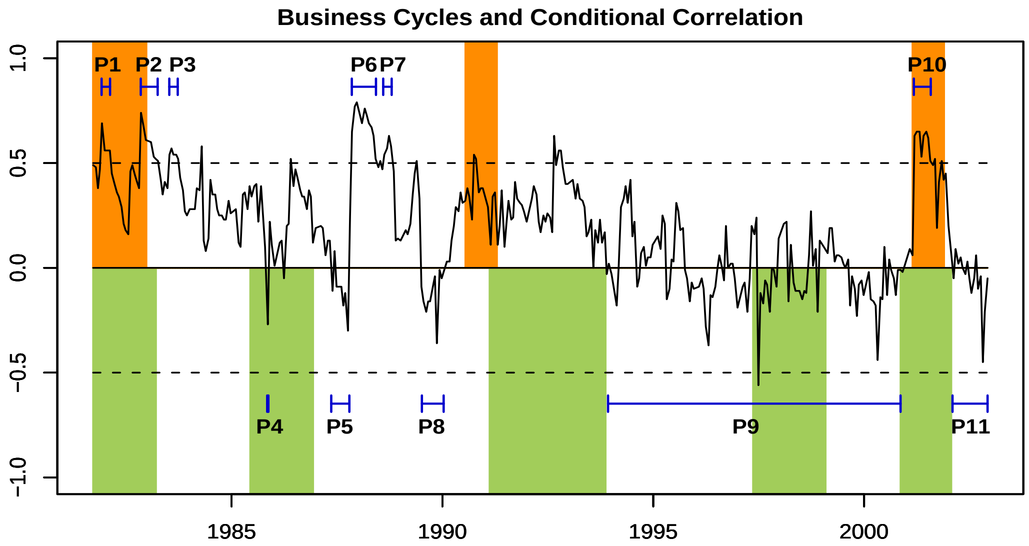
The plots present the reprojected conditional correlation for the $\mathbb{IQ}(3;1,1,1)$ model against the projected conditional correlation. The reprojected data are represented by dashed lines, whereas the projected data are represented by solid line.

Figure 18: **Projected Conditional Correlation and Periods**



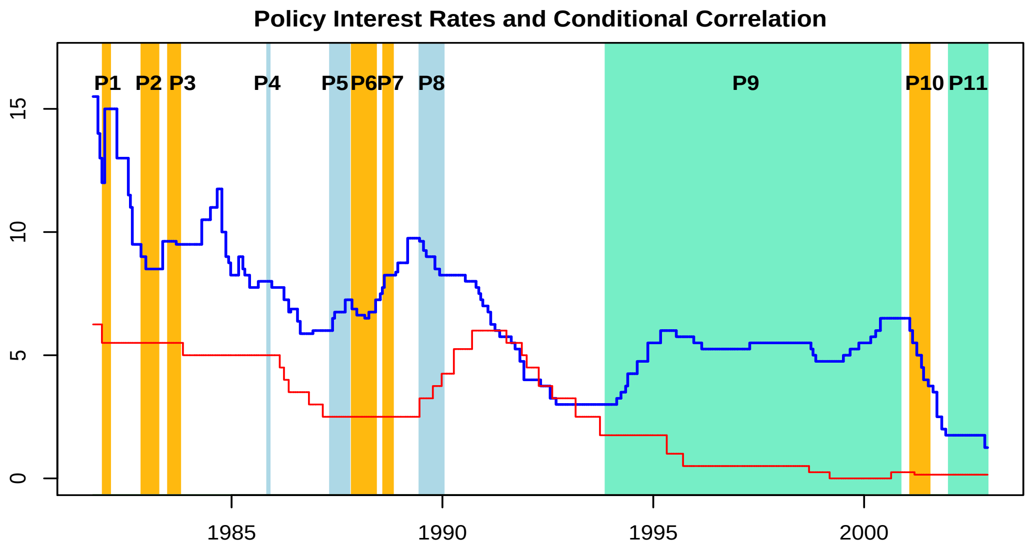
The plots present the projected conditional correlation between the six-month Eurodollar and Euroyen yields. Shaded areas represent each of the 11 periods; P1(December 1981-January 1982), P2(November 1982-March 1983), P3(June 1983-September 1983), P4(October 1985), P5(April 1987-September 1987), P6(November 1987-May 1988), P7(August 1988-October 1988), P8(June 1989-November 1989), P9(November 1993-November 2000), P10(February 2001-July 2001), and P11(January 2002-November 2002)

Figure 19: **Projected Conditional Correlation and Business Cycles**



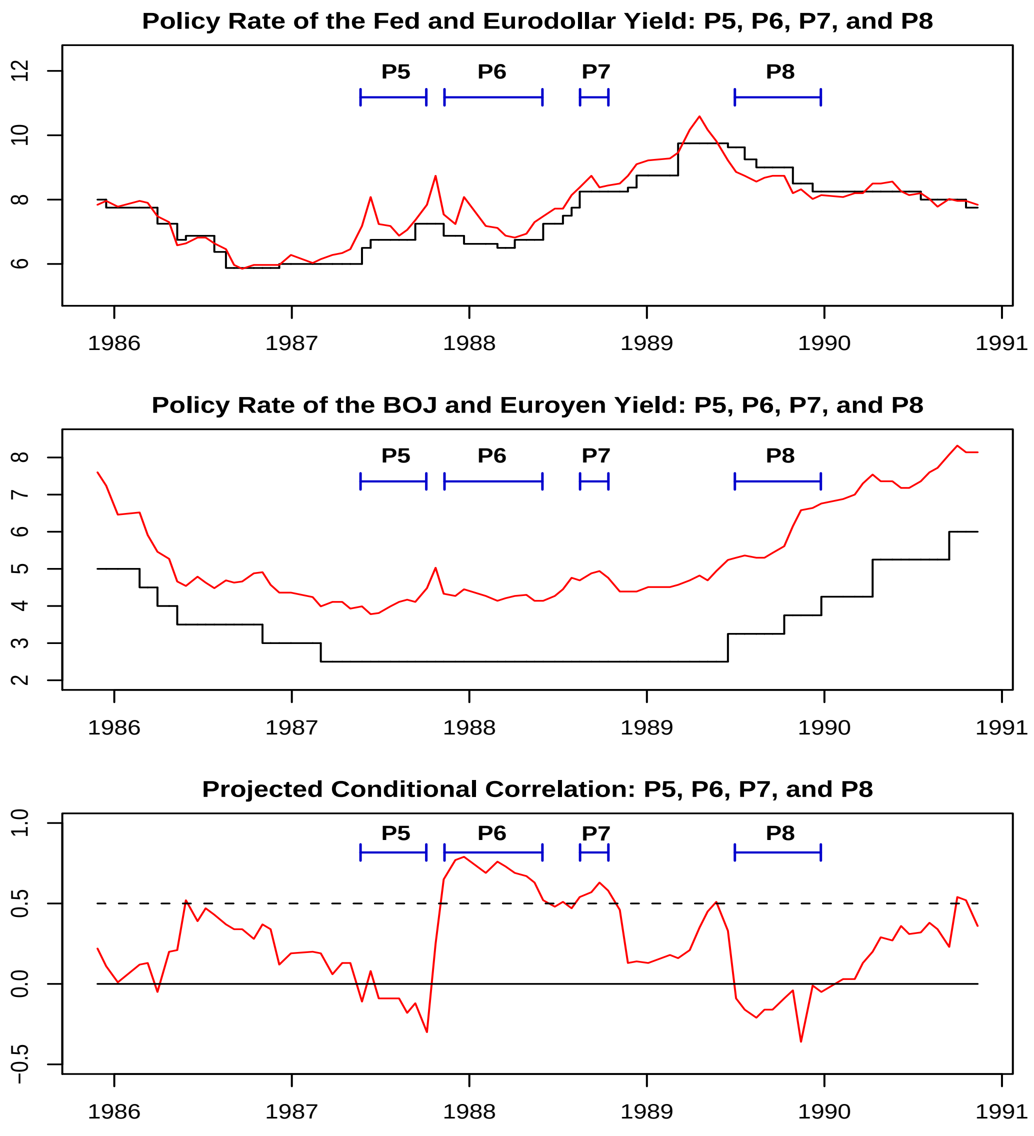
The plots present the projected conditional correlation between the six-month Eurodollar and Euroyen yields, business cycles of the U.S. and Japan, and each of the 11 periods. The upper shaded area represents recessions in the U.S. defined by the NBER (National Bureau of Economic Research) while the rest periods are defined as boom. The bottom shaded area indicates recession in Japan defined by the ESRI (Economic and Social Research Institute of the Cabinet Office of the Japanese government). According to the NBER reference business cycle, there were four recessions in the U.S. since 1980: from January 1980 to July 1980, from July 1981 to November 1982, from July 1990 to March 1991, and from March 2001 to November 2001. According to the ESRI's official business cycle dating, there were five recessions in Japan since 1980: from February 1980 to February 1983, from June 1985 to November 1986, from February 1991 to October 1993, from May 1997 to January 1999, and from November 2000 to January 2002.

Figure 20: **Projected Conditional Correlation and Monetary Policies**



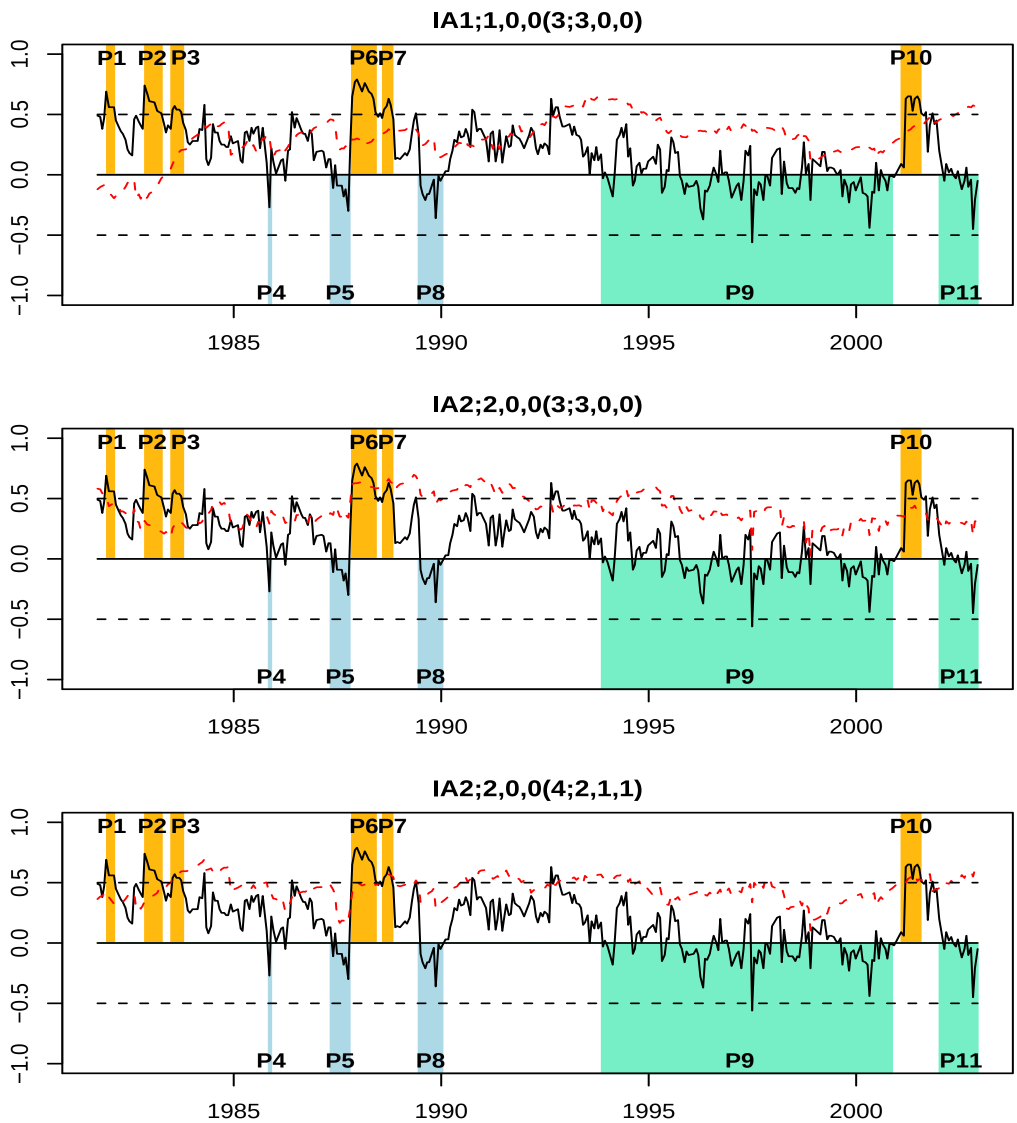
The plots presents the projected conditional correlation between the six-month Eurodollar and Euroyen yields and the policy interest rates of the Fed (Federal Reserve System) and the BOJ (Bank of Japan).

Figure 21: **Episodes of P5, P6, P7, and P8**



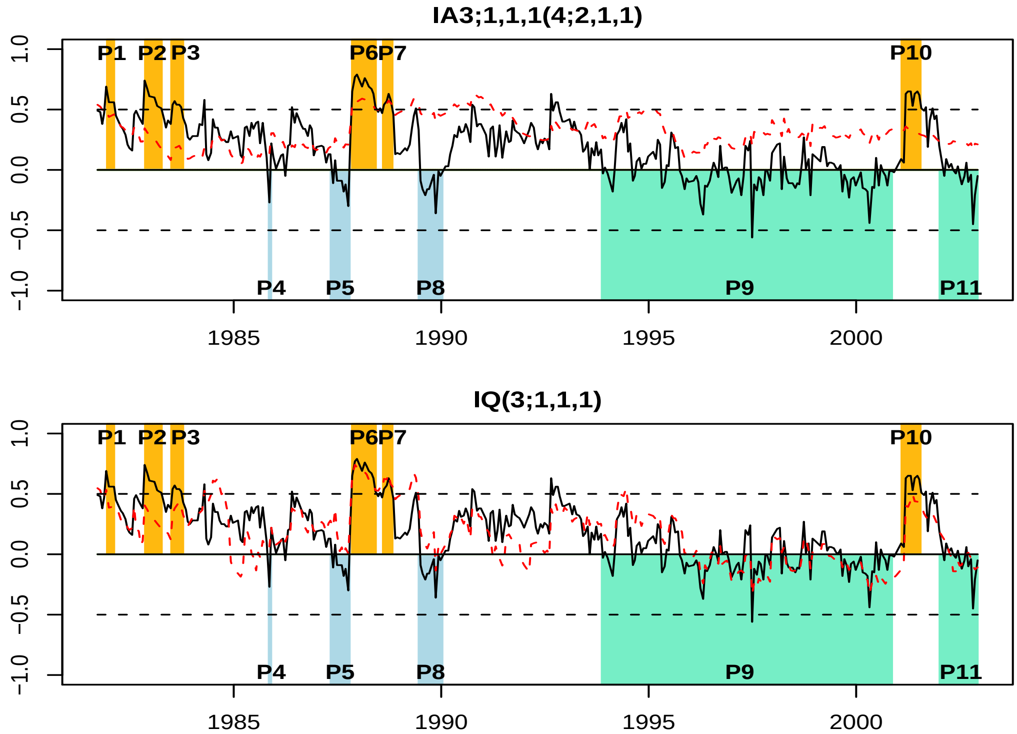
The plots presents the observed six-month Eurodollar and Euroyen yields, the projected conditional correlation between the two yields, and the policy interest rates of the Fed and the BOJ during P5(April 1987-September 1987), P6(November 1987-May 1988), P7(August 1988-October 1988), and P8(June 1989-November 1989).

Figure 22: **Reprojected Conditional Correlation: Eurodollar Yield by Euroyen Yield**



The plots present the reprojected conditional correlation for the $\mathbb{IA}_{1;1,0,0}(3;3,0,0)$, $\mathbb{IA}_{2;2,0,0}(3;3,0,0)$, and $\mathbb{IA}_{2;2,0,0}(4;2,1,1)$ models against the projected conditional correlation. The reprojected data are represented by dashed lines, whereas the projected data are represented by solid line.

Figure 23: **Reprojected Conditional Correlation: Eurodollar Yield by Euroyen Yield**



The plots present the reprojected conditional correlation for the $\mathbb{IA}_{3;1,1,1}(4;2,1,1)$ and $\mathbb{IQ}(3;1,1,1)$ models against the projected conditional correlation. The reprojected data are represented by dashed lines, whereas the projected data are represented by solid line.

References

Ahn, Dong-Hyun, 2004, Common Factors and Local Factors: Implications for Term Structures and Exchange Rates, *Journal of Financial and Quantitative Analysis*, 39, 69-102.

Ahn, Dong-Hyun, and Bin Gao, 1999, A Parametric Nonlinear Model of Term Structure Dynamics, *Review of Financial Studies*, 12, 721-762.

Ahn, Dong-Hyun, and Bin Gao, 2003, Locally Complete Markets, Exchange Rates and Currency Options, *Review of Derivatives Research*, 6, 5-26.

Ahn, Dong-Hyun, Robert F. Dittmar, and A. Ronald Gallant, 2002, Quadratic Term Structure Models: Theory and Evidence, *Review of Financial Studies*, 15, 243-288.

Ahn, Dong-Hyun, Robert F. Dittmar, A. Ronald Gallant, and Bin Gao, 2003, Purebred or Hybrid?: Reproducing the Volatility in Term Structure Dynamics, *Journal of Econometrics*, 116, 147-180.

Aït-Sahalia, Yacine, 1996, Nonparametric Pricing of Interest Rate Derivative Securities, *Econometrica*, 64, 527-560.

Ang, Andrew, and Geert Bekaert, 2002, Regime Switches in Interest Rates, *Journal of Business and Economic Statistics*, 20, 163-182.

Ang, Andrew, Jean Boivin, Sen Dong, and Rudy Loo-Kung, 2009, Monetary Policy Shifts and the Term Structure, *Working Paper*, *Columbia University*.

Ang, Andrew, and Joseph Chen, 2002, Asymmetric Correlations of Equity Portfolios, *Journal of Financial Economics*, 63, 443-494.

Ang, Andrew, and Monica Piazzesi, 2003, A No-Arbitrage Vector Autoregression of Term Structure Dynamics with Macroeconomic and Latent Variables, *Journal of Monetary Economics*, 50, 745-787.

Backus, David K., Silverio Foresi, and Chris I. Telmer, 2001, Affine Term Structure Models and the Forward Premium Anomaly, *Journal of Finance*, 56, 279-304.

Bae, Kee-Hong, G. Andrew Karolyi, and René M. Stulz, 2003, A New Approach to Measuring Financial Contagion, *Review of Financial Studies*, 16, 717-763.

Baig, Taimur, and Ilan Goldfajn, 1999, Financial Market Contagion in the Asian Crisis, *IMF Staff Papers*, 46, 167-195.

Bakshi, Gurdip S., and Zhiwu Chen, 1997, Equilibrium Valuation of Foreign Exchange Claims, *Journal of Finance*, 52, 799-826.

Bansal, Ravi, and Hao Zhou, 2002, Term Structure of Interest Rates with Regime Shifts, *Journal of Finance*, 57, 1997-2043.

Benati, Luca, 2006, Affine Term Structure Models for the Foreign Exchange Risk Premium, *Working Paper*, *Bank of England*.

Bergstrom, Gary L., and Roy D. Henrikson, 1981, Prediction of the International Equity Market Covariance Structure, *CRSP Proceedings*, 26, 131-144.

Bernanke, Ben, and Mark Gertler, 1999, Monetary Policy and Asset Price Volatility, *Federal Reserve Bank of Kansas City Economic Review*, 84, 17-52.

Bernanke, Ben, and Frederic S. Mishkin, 1992, Central Bank Behavior and the Strategy of Monetary Policy: Observations from Six Industrialized Countries, *NBER Macroeconomics Annual*, 7, 183-228.

Board of Governors of the Federal Reserve System, 1986, Monetary Policy Report to the Congress, *Federal Reserve Bulletin*, September, 603-617.

Board of Governors of the Federal Reserve System, 1987a, Monetary Policy Report to the Congress, *Federal Reserve Bulletin*, April, 239-254.

Board of Governors of the Federal Reserve System, 1987b, Monetary Policy Report to the Congress, *Federal Reserve Bulletin*, August, 633-646.

Board of Governors of the Federal Reserve System, 1988, Monetary Policy Report to the Congress, *Federal Reserve Bulletin*, March, 151-164.

Bollerslev, Tim, 1990, Modelling the Coherence in Short-Run Nominal Exchange Rates: A Multivariate Generalized ARCH Model, *Review of Economics and Statistics*, 72, 498-505.

Bollerslev, Tim, Robert F. Engle, and Jeffery M. Wooldridge, 1988, A Capital Asset Pricing Model with Time-Varying Covariances, *Journal of Political Economy*, 96, 116-131.

Brandt, Michael W., and Pedro Santa-Clara, 2002, Simulated Likelihood Estimation of Diffusions with an Application to Exchange Rate Dynamics in Incomplete Markets *Journal of Financial Economics*, 63, 161-210.

Brennan, Michael J., and Yihong Xia, 2006, International Capital Markets and Foreign Exchange Risk, *Review of Financial Studies*, 19, 753-795.

Brown, Stephen J., 1989, The Number of Factors in Security Returns, *Journal of Finance*, 44, 1247-1262.

Brusco, Sandro, and Fabio Castiglionesi, 2007, Liquidity Coinsurance, Moral Hazard, and Financial Contagion, *Journal of Finance*, 62, 2275-2302.

Calvo, Guillermo A., 1999, Contagion in Emerging Markets: When Wall Street is a Carrier, *Working Paper*, *University of Maryland*.

Campbell, Sean D., and Francis X. Diebold, 2009, Stock Returns and Expected Business Conditions: Half a Century of Direct Evidence *Journal of Business and Economic Statistics*, 27, 266-278.

Canova, Fabio, Matteo Ciccarelli, and Eva Ortega, 2007, Similarities and Convergence in G-7 Cycles, *Journal of Monetary Economics*, 54, 850-878.

Cappiello, Lorenzo, Robert F. Engle, and Kevin Sheppard, 2006, Asymmetric Dynamics in the Correlations of Global Equity and Bond Returns, *Journal of Financial Econometrics*, 4, 537-572.

Chernozhukov, Victor, and Han Hong, 2003, An MCMC Approach to Classical Estimation, *Journal of Econometrics*, 115, 293-346.

Claessens, Stijn, and Kristin J. Forbes (eds.), 2001, *International Financial Contagion*, Kluwer Academic Publishers, Massachusetts.

Claessens, Stijn, M. Ayhan Kose, and Marco E. Terrones, 2008, What Happens during Recessions, Crunches and Busts? *IMF Working Paper*, 08-274.

Constantinides, George M., 1992, A Theory of the Nominal Term Structure of Interest Rates, *Review of Financial Studies*, 5, 531-552.

Cook, Timothy, 1989, Determinants of the Federal Funds Rate: 1979-1982, *Federal Reserve Bank of Richmond Economic Review*, January/February, 3-19.

Dai, Qiang, and Kenneth J. Singleton, 2000, Specification Analysis of Affine Term Structure Models, *Journal of Finance*, 55, 1943-1978.

Dai, Qiang, and Kenneth J. Singleton, 2003, Term Structure Dynamics in Theory and Reality, *Review of Financial Studies*, 16, 631-678.

Danielsson, Jon, and Casper G. De Vries, 2000, Value-at-Risk and Extreme Returns, *Annales d'Économie et de Statistique*, 60, 239-270.

Diebold, Francis X., and Marc Nerlove, 1989, The Dynamics of Exchange Rate Volatility: A Multivariate Latent Factor ARCH Model, *Journal of Applied Econometrics*, 4, 1-21.

Doyle, Brian M., and Jon Faust, 2005, Breaks in the Variability and Co-Movement of G-7 Economic Growth, *Review of Economics and Statistics*, 87, 721-740.

Duffee, Gregory R., 2002, Term Premia and Interest Rate Forecasts in Affine Models, *Journal of Finance*, 57, 405-443.

Duffee, Gregory R., 2009, Information in (and not in) the Term Structure, *Working Paper*, *Johns Hopkins University*.

Duffie, Darrell, and Rui Kan, 1996, A Yield-Factor Model of Interest Rates, *Mathematical Finance*, 6, 379-406.

Egorov, Alexei V., Haitao Li, and David Ng, 2009, A Tale of Two Yield Curves: Modeling the Joint Term Structure of Dollar and Euro Interest Rates, *Forthcoming*, *Journal of Econometrics*.

Engle, Robert F., 2002, Dynamic Conditional Correlation - A Simple Class of Multivariate GARCH Models, *Journal of Business and Economic Statistics*, 20, 339-350.

Engle, Robert F., and Kenneth F. Kroner, 1995, Multivariate Simultaneous GARCH, *Econometric Theory*, 11, 122-150.

Engle, Robert F., Victor K. Ng, and Michael Rothschild, 1990, Asset Pricing with a Factor ARCH Covariance Structure: Empirical Estimates for Treasury Bills, *Journal of Econometrics*, 45, 213-237.

Erb, Claude B., Campbell R. Harvey, and Tadas E. Viskanta, 1994, Forecasting International Equity Correlations, *Financial Analysts Journal*, November/December, 32-45.

Eun, Cheol S, and Bruce G. Resnick, 1984, Estimating the Correlation Structure of International Share Prices, *Journal of Finance*, 39, 1311-1324.

Fama, Eugene F., 1986, Term Premiums and Default Premiums in Money Markets, *Journal of Financial Economics*, 17, 175-l96.

Fama, Eugene F., and Kenneth R. French, 1989, Business Conditions and Expected Returns on Stocks and Bonds, *Journal of Financial Economics*, 25, 23-49.

Ferguson, Roger W., 2005, Recessions and Recoveries Associated with Asset-Price Movements: What Do We Know?, Remarks at the Stanford Institute for Economic Policy Research, Stanford, California, January 12. (www.federalreserve.gov/BOARDDOCS/Speeches/2005/20050112/default.htm)

Forbes, Kristin, and Roberto Rigobon, 2001, Measuring Contagion: Conceptual and Empirical Issues, in Stijn Claessens and Kristin J. Forbes, (eds.), *International Financial Contagion*, Kluwer Academic Publishers, Massachusetts.

Gallant, A. Ronald, and Jonathan R. Long, 1997, Estimating Stochastic Differential Equations Efficiently by Minimum Chi-Squared, *Biometrika*, 84, 125-141.

Gallant, A. Ronald, and George Tauchen, 1996, Which Moments to Match, *Econometric Theory*, 12, 657-681.

Gallant, A. Ronald, and George Tauchen, 1998, Reprojecting Partially Observed Systems with Application to Interest Rate Diffusions, *Journal of the American Statistical Association*, 93, 10-24.

Gallant, A. Ronald, and George Tauchen, 2002, Efficient Method of Moments, *Working Paper*, *Duke University*.

Gallant, A. Ronald and George Tauchen, 2009, SNP: A Program for Nonparametric Time Series Analysis, Version 9.0, User's Guide, *Department of Economics*, *Duke University*, http://econ.duke.edu/webfiles/arg/snp/.

Gallant, A. Ronald and George Tauchen, 2010a, Simulated Score Methods and Indirect Inference for Continuous-Time Models, in Yacine Aït-Sahalia and Lars Peter Hansen, (eds.), *Handbook of Financial Econometrics*, Volume 1 – Tools and Techniques, Elsevier/North-Holland, Amsterdam, 427-478.

Gallant, A. Ronald, and George Tauchen, 2010b, EMM: A Program for Efficient Method of Moments Estimation, Version 2.6, User's Guide, *Department of Economics*, *Duke University*, http://econ.duke.edu/webfiles/arg/emm/.

Goodfriend, Marvin, 1993, Interest Rate Policy and the Inflation Scare Problem: 1979-1992, *Federal Reserve Bank of Richmond Economic Quarterly*, 79, 1-23.

Goodfriend, Marvin, 2002, The Phases of U.S. Monetary Policy: 1987 to 2001, *Federal Reserve Bank of Richmond Economic Quarterly*, 88, 1-17.

Greenspan, Alan, 2004, Risk and Uncertainty in Monetary Policy, *American Economic Review Papers and Proceedings*, 94, 33-40.

Gregory, Allan W., Allen C. Head, and Jacques Raynauld, 1997, Measuring World Business Cycles, *International Economic Review*, 38, 677-701.

Hamada, Koichi, and Yasushi Okada, 2009, Monetary and International Factors behind Japan's Lost Decade, *Journal of the Japanese and International Economies*, 23, 200-219.

Hayashi, Fumio, and Edward C. Prescott, 2002, The 1990s in Japan: A Lost Decade, *Review of Economic Dynamics*, 5, 206-235.

Heathcote, Jonathan, and Fabrizio Perri, 2004, Financial Globalization and Real Regionalization, *Journal of Economic Theory*, 119, 207-243.

Helbling, Thomas, and Tamin Bayoumi, 2003, Are They All in the Same Boat? The 2000-2001 Growth Slowdown and the G-7 Business Cycle Linkages, *IMF Working Paper*, 03-46.

Hester, Donald D., 2008, *The Evolution of Monetary Policy and Banking in the U.S.*, Springer-Verlag, Berlin.

IMF, 2001, International Linkages: Three Perspectives, *World Economic and Financial Surveys*, IMF, Washington, D.C.

Inci, Ahmet Can, 2007, U.S.-Swiss Term Structures and Exchange Rate Dynamics, *Global Finance Journal*, 18, 270-288.

Inci, Ahmet Can, and Biao Lu, 2004, Exchange Rates and Interest Rates: Can Term Structure Models Explain Currency Movements?, *Journal of Economic Dynamics and Control*, 28, 1595-1624.

Ito, Takatoshi, 2004, The Yen and the Japanese Economy, in C. Fred Bergsten and John Williamson, (eds.), *Dollar Adjustment: How Far? Against What?*, Institute for International Economics, Washington, D.C.

Ito, Takatoshi, 2009, Great Inflation and Central Bank Independence in Japan, *Working Paper*, *University of Tokyo.*

Ito, Takatoshi, and Frederic S. Mishkin, 2004, Two Decades of Japanese Monetary Policy and the Deflation Problem, *Working Paper*, *NBER.*

Joslin, Scott, Marcel Priebsch, and Kenneth J. Singleton, 2009, Risk Premium Accounting in Macro-Dynamic Term Structure Models, *Working Paper*, *Stanford University.*

Kaminsky, Graciela L., and Sergio L. Schmukler, 1999, What Triggers Market Jitters?: A Chronicle of the Asian Crisis, *Journal of International Money and Finance*, 18, 537-560.

Kaplanis, Evi C., 1988, Stability and Forecasting of the Comovement Measures of International Stock Market Returns, *Journal of International Money and Finance*, 7, 63-75.

King, Mervyn A., and Sushil Wadhwani, 1990, Transmission of Volatility between Stock Markets, *Review of Financial Studies*, 3, 5-33.

Kordres, Laura E., and Matthew Pritsker, 2002, A Rational Expectations Model of Financial Contagion, *Journal of Finance*, 57, 769-799.

Kose, M. Ayhan, Christopher Otrok, and Charles H. Whiteman, 2005, Understanding the Evolution of World Business Cycles, *IMF Working Paper*, 05-211.

Kose, M. Ayhan, Christopher Otrok, and Charles H. Whiteman, 2008, Understanding the Evolution of World Business Cycles, *Journal of International Economics*, 75, 110-130.

Kyle, Albert S., and Wei Xiong, 2001, Contagion as a Wealth Effect, *Journal of Finance*, 56, 1401-1440.

Leippold, Markus, and Liuren Wu, 2007, Design and Estimation of Multi-Currency Quadratic Models, *Review of Finance*, 11, 167-207.

Litterman, Robert, and José Scheinkman, 1991, Common Factors Affecting Bond Returns, *Journal of Fixed Income*, 1, 54-61.

Longin, François M., 1996, The Asymptotic Distribution of Extreme Stock Market Returns, *Journal of Business*, 69, 383-408.

Longin, François M., and Bruno Solnik, 1995, Is the Correlation in International Equity Returns Constant: 1960-1990? *Journal of International Money and Finance*, 14, 3-26.

Longin, François M., and Bruno Solnik, 2001, Extreme Correlation of International Equity Markets, *Journal of Finance*, 56, 649-676.

Ludvigson, Sydney C., and Serena Ng, 2009, Macro Factors in Bond Risk Premia, *Review of Financial Studies*, 22, 5027-5067.

Mishkin, Frederic S., 2008, How Should We Respond to Asset Price Bubbles?, Member of the Board of Governors of the Federal Reserve System, Speech at the Wharton Financial Institutions Center and Oliver Wyman Institute's Annual Financial Risk Roundtable, Philadelphia, Pennsylvania, May 15. (www.federalreserve.gov/newsevents/speech/mishkin20080-515a.htm)

Mosburger, Georg, and Paul Schneider, 2005, Modelling International Bond Markets with Affine Term Structure Models, *Working Paper*, *University of Vienna.*

Mussa, Michael L., 1994, U.S. Monetary Policy in the 1980s, in Martin Feldstein, (eds.), *American Economic Policy in the 1980s*, University of Chicago Press, Chicago.

Newey, Whitney K., and Kenneth D. West, 1987, A Simple, Positive Semi-Definite, Heteroskedasticity and Autocorrelation Consistent Covariance Matrix, *Econometrica*, 55, 703-708.

Nielson, Lars T., and Jesús Saá-Requejo, 1993, Exchange Rate and Term Structure Dynamics and the Pricing of Derivative Securities, *Working Paper*, *INSEAD.*

Obstfeld, Maurice, 2009, Time of Troubles: The Yen and Japan's Economy, 1985-2008, *Working Paper*, it University of California, Berkeley.

Okina, Kunio, Masaaki Shirakwa, and Shigenori Shiratsuka, 2001, The Asset Price Bubble and Monetary Policy: Japan's Experience in the Late 1980s and the Lessons, *Monetary and Economic Studies*, 19, 395-450.

Palandri, Alessandro, 2009, Sequential Conditional Correlations: Inference and Evaluation, *Journal of Econometrics*, 153, 122-132.

Rudebusch, Glenn D., and Tao Wu, 2008, A Macro-Finance Model of the Term Structure, Monetary Policy and the Economy, *Economic Journal*, 118, 906-926.

Saá-Requejo, Jesús, 1993, The Dynamics and the Term Structure of Risk Premia in Foreign Exchange Markets, *Working Paper*, *INSEAD*.

Schwarz, Gideon, 1978, Estimating the Dimension of a Model, *Annals of Statistics*, 6, 461-464.

Shinotsuka, Eiko, 2000, Japan's Economy and the Role of Bank of Japan, Member of the Policy Board of the Bank of Japan, Speech to the Japan Society of Monetary Economics, May 27. (www.boj.or.jp/en/type/press/koen/ko0007b.htm)

Shiratsuka, Shigenori, 2005, The Asset Price Bubble in Japan in the 1980s: Lessons for Financial and Macroeconomic Stability, *BIS Papers*, 21, 42-62.

Singleton, Kenneth J., 1994, Persistence of International Interest Rate Correlation, *Working Paper*, *Stanford University*.

Stambaugh, Robert F., 1988, The Information in Forward Rates: Implications for Models of the Term Structure, *Journal of Financial Economics*, 21, 41-70.

Starica, Catalin, 1999, Multivariate Extremes for Models with Constant Conditional Correlations, *Journal of Empirical Finance*, 6, 515-553.

Stock, James H., and Mark W. Watson, 2005, Understanding Changes in International Business Cycle Dynamics, *Journal of the European Economic Association*, 3, 968-1006.

Straetmans, Stefan, 1998, Extreme Financial Returns and Their Comovements, *Tinbergen Institute Research Series*, Ph.D. thesis no.181.

Tauchen, George, 1998, The Objective Function of Simulation Estimators near the Boundary of the Unstable Region of the Parameter Space, *Review of Economics and Statistics*, 80,389-398.

Taylor, John B, 1998, Monetary Policy and the Long Boom, *Federal Reserve Bank of St. Louis Review*, 80, 3-12.

Ueda, Kazuo, 1991, Japanese Monetary Policy during 1970-1990: Rules or Discretion?, *Working Paper*, *University of Tokyo*.

Ueda, Kazuo, 1993, A Comparative Perspective on Japanese Monetary Policy: Short-Run Monetary Control and the Transmission Mechanism, in Kenneth J. Singleton, (eds.), *Japanese Monetary Policy*, University of Chicago Press, Chicago.

Ueda, Kazuo, 2000, Member of the Policy Board of the Bank of Japan, Remarks at the Geneva Conference on "Asset Price Inflation: What to Do about It?" organized by the International Center for Monetary and Banking Studies and the CEPR on May 5. (www.boj.or.jp/en/type/press/koen/ko0005a.htm)

Von Furstenberg, George M., and Bang N. Jeon, 1989, International Stock Price Movements: Links and Messages, *Brookings Papers on Economic Activity*, 1, 125-179.

Wall, Howard J., 2006, Regional Business Cycle Phases in Japan, *Working Paper*, *Federal Reserve Bank of St. Louis*.

Watanabe, Toshiaki, 2003, Measuring Business Cycle Turning Points in Japan with a Dynamic Markov Switching Factor Model, *Monetary and Economic Studies*, 21, 35-68.

Yamaguchi, Yutaka, 2000, Thinking behind Current Monetary Policy, Deputy Governor of the Bank of Japan, Remarks at the Japan National Press Club on August 4. (www.boj.or.jp/en/type/press/koen/ko0008a.htm)

Yuan, Kathy, 2005, Asymmetric Price Movements and Borrowing Constraints: A Rational Expectations Equilibrium Model of Crises, Contagion, and Confusion, *Journal of Finance*, 60, 379-411.