In-Class Permit Trading Exercise
Environmental Economics
Professor A. Ronald Gallant
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During class #6 on February 9, 2009, we will have an in-class permit trading exercise. As preparation for that exercise, please review the information below and make sure that you are comfortable with all the calculations and analysis performed. When we discuss the exercise in class, I will assume you have mastered this basic analysis. You should think about how the firms would behave in an environment with tradable emission permits.

1 Basics

- **Participants:** Each team will be assigned a number. The exercise assumes there is an even number of teams. Each team represents a firm.

- **Output market:** The firms operate in the same country. Each firm produces the same product, which is traded on a global scale. Production in the firms’ home country is small relative to global production, so the firms take the market price of 40 for their output as given.

- **Production:** Each firm’s production cost associated with $Q$ units of output is $C(Q) = Q$ up to a capacity of 20 (all firms have the same cost function and same capacity constraint). Marginal cost is constant at 1.

- **Pollution:** Each unit of output produces one unit of pollution.
• **Pollution abatement:** A firm can reduce its emissions by investing in pollution abatement. For odd-numbered firms, the cost of reducing pollution by $A$ units is $C_{\text{odd}}^A(A) = 3A^2$. For even-numbered firms, the cost is $C_{\text{even}}^A(A) = A^2$. Thus, odd-numbered firms have relatively higher abatement costs than even-numbered firms. Marginal abatement costs are $6A$ for odd-numbered firms and $2A$ for even-numbered firms.

### 1.1 No regulation

To fix ideas, consider the case in which there is no environmental regulation. In the absence of environmental regulation, each firm chooses its output and pollution abatement to maximize its profit (reductions cannot exceed the total amount of pollution, which is $Q$, so it must be that $A \leq Q$). Thus, odd-numbered firms solve:

$$\max_{Q,A} 40Q - Q - 3A^2$$

and even-numbered firms solve:

$$\max_{Q,A} 40Q - Q - A^2.$$  

Of course, in this case, the firms optimally choose $A = 0$ and $Q = 20$, i.e., firms do not invest in any pollution abatement and they operate at capacity. Profit is $40 \cdot 20 - 20 = 780$.

### 1.2 Emission cap

Suppose the government of the firms’ home country institutes a cap on emissions of 10 units, with each unit of pollution in excess of this limit fined at a rate of 70/unit. A firm’s pollution associated with output $Q$ is $Q - A$. To avoid the fine, a firm must have $Q - A \leq 10$. Thus, firms choose $A = Q - 10$.\(^1\)

Using $A = Q - 10$, an odd-numbered firm’s profit maximization problem is:

$$\max_Q 40Q - Q - 3(Q - 10)^2.$$  

\(^1\)Because the capacity constraint is 20, the maximum pollution abatement a firm would choose is $A = 10$. The marginal cost of abatement for an odd-numbered firm when $A = 10$ is 60, which is less than the fine of 70, so it is always optimal for the firms to abate rather than pay the fine.
Differentiating with respect to $Q$, we get

$$40 - 1 - 6(Q^* - 10) = 0,$$

which implies

$$Q^* = \frac{99}{6} = 16.50 \text{ and } A^* = 6.50.$$  

Thus, odd-numbered firms reduce output below their unregulated levels. Profit is 516.75,\(^2\) approximately one-third less than the unregulated profit.

For even-numbered firms, it is optimal to produce at capacity and to invest in 10 units of abatement. Profit is 680 (even-numbered firms benefit from having lower abatement costs).

Intuitively, an additional unit of output requires an additional unit of pollution abatement, so an even-numbered firm’s total marginal cost is $1 + 2(Q - 10)$, which is equal to 21 at $Q = 20$. Even-numbered firms would like to continue to increase their output until their total marginal cost is equal to the output price, but they hit their capacity constraints before that happens.

\(^2\)Profit is $40Q^* - Q^* - 3A^*$. 