THE PENNSYLVANIA STATE UNIVERSITY
Department of Economics

Economics 501 | Gallant
Homework 10 | Fall 2014
Nov. 11

1. Plot the empirical distribution function of the numbers 0.71, 0.73, 1.57, 1.61, 0.02, 0.7, 0.67, 1.1, 1.8, and 0.76.

2. Let f_X be a density function, which may be either discrete or continuous, that is symmetric about zero and let $Y = X I_{[-B,B]}(X)$. Show that $\mathcal{E}Y = 0$.

3. Let X_i be independently and identically distributed with finite variance. Show that $S_n^2 = (n-1)^{-1}\sum_{i=1}^n (X_i - \bar{X}_n)^2$ where $\bar{X}_n = n^{-1}\sum_{i=1}^n X_i$ converges almost surely to $\text{Var}(X)$.

4. In a common valuation, oral ascending auction with n bidders, the winner pays the second largest value in a random sample $X_1, \cdots, X_n$ from the common valuation distribution $F_X(x)$. Derive the distribution of the winning bid.
   Hint: If there are two bidders and Y denotes the winning bid then $F_Y(y) = P(X_1 \le y,\, X_2 \le y) + P(X_1 \le y,\, y < X_2) + P(X_2 \le y,\, y < X_1)$.

5. Let U and V be independent uniform random variables. Show that

$$\begin{aligned} X &= \cos(2\pi U)\sqrt{-2\log V} \\ Y &= \sin(2\pi U)\sqrt{-2\log V} \end{aligned}$$

   are independent normal random variables.