THE PENNSYLVANIA STATE UNIVERSITY
Department of Economics

Economics 501 Gallant
Homework 7 Fall 2014
Due Oct. 14

1. Draw a diagram upon which are superimposed the sets $(-\infty, b_x] \times (-\infty, b_y]$, $(-\infty, a_x] \times (-\infty, b_y]$, $(-\infty, b_x] \times (-\infty, a_y]$, and $(-\infty, a_x] \times (-\infty, a_y]$. Mark the four points (a_x, a_y), (b_x, a_y), (a_x, b_y), and (b_x, b_y) on the diagram. Use Proposition 1.1 to show that $P(a_x < X \leq b_x, a_y < Y \leq b_y) = F_{X,Y}(b_x, b_y) - F_{X,Y}(a_x, b_y) - F_{X,Y}(b_x, a_y) + F_{X,Y}(a_x, a_y)$.

2. If X and Y are independent random variables, show that $F_{X,Y}(b_x, b_y) - F_{X,Y}(a_x, b_y) - F_{X,Y}(b_x, a_y) + F_{X,Y}(a_x, a_y) = [F_X(b_x) - F_X(a_x)][F_Y(b_y) - F_Y(a_y)]$.

3. Let X be continuous random variable with distribution function $F_X(x)$ and let Y be a continuous random variable with distribution $F_Y(y)$. Assume that both F_X and F_Y are strictly increasing.

   (a) What is the transformation $g(x)$ such that the random variable $W = g(X)$ has the uniform distribution.

   (b) What is the transformation $g(x)$ such that the random variable $W = g(X)$ is distributed as F_Y.

4. Suppose that $f_X(x) = (1/\sigma) f_Z[(x - \mu)/\sigma]$ where $f_Z(z)$ is a density with mean 0 and standard deviation 1. What is the mean and variance of the random variable X.

5. For each density f_X, support $\mathcal{X}$, and transformation $Y = g(X)$ listed below find the density f_Y and support $\mathcal{Y}$ of the random variable Y. Check your work by verifying that $\int_{\mathcal{Y}} f_Y(y)\, dy = 1$.

   (a) $f_X(x) = 42x^5(1-x),\ \mathcal{X} = \{x : 0 < x < 1\},\ Y = X^3$.

   (b) $f_X(x) = 5e^{-5x},\ \mathcal{X} = \{x : 0 < x < \infty\},\ Y = 2X + 1$.

   (c) $f_X(x) = (2\pi)^{-1/2} e^{-x^2/2};\mathcal{X} = \{x : -\infty < x < \infty\},\ Y = 2X + 1$.

(d) $f_X(x) = (2\pi)^{-1/2} e^{-x^2/2}$; $\mathcal{X} = \{x : -\infty < x < \infty\}$, $Y = e^X$.

(e) $f_X(x) = (2\pi)^{-1/2} e^{-x^2/2}$; $\mathcal{X} = \{x : -\infty < x < \infty\}$, $Y = |X|$.