THE PENNSYLVANIA STATE UNIVERSITY
Department of Economics

Economics 501 | Gallant
Homework 4 | Fall 2014
Due Sept. 23

1. A pair of correlated, six-sided dice are tossed. The random variable X denotes the first toss and the random variable Λ denotes the second; realizations of these tosses are pairs $(x, \lambda) \in \{1, 2, 3, 4, 5, 6\} \times \{1, 2, 3, 4, 5, 6\}$. The random variable $D = X - \Lambda$ is the difference. The preimages of D are shown in the first column below and the probabilities $P(D = d)$ are shown in the third column. Fill in the correct entries for the third and fourth columns.

Preimage	d	$P(D = d)$	$P(D = d \mid \Lambda = 1)$	$P(D = d \mid \Lambda = 2)$
$C_{-5} = \{(1, 6)\}$	-5	0	-	-
$C_{-4} = \{(1, 5), (2, 6)\}$	-4	0	-	-
$C_{-3} = \{(1, 4), (2, 5), (3, 6)\}$	-3	0	-	-
$C_{-2} = \{(1, 3), (2, 4), (3, 5), (4, 6)\}$	-2	0	-	-
$C_{-1} = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)\}$	-1	4/18	-	-
$C_0 = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$	0	10/18	-	-
$C_1 = \{(2, 1), (3, 2), (4, 3), (5, 4), (6, 5)\}$	1	4/18	-	-
$C_2 = \{(3, 1), (4, 2), (5, 3), (6, 4))\}$	2	0	-	-
$C_3 = \{(4, 1), (5, 2), (6, 3)\}$	3	0	-	-
$C_4 = \{(5, 1), (6, 2)\}$	4	0	-	-
$C_5 = \{(6, 1)\}$	5	0	-	-

2. If A and B are subsets of $\mathcal{X}$, and $A_1, A_2, \ldots$ is a sequence of subsets from $\mathcal{X}$, show that the inverse image satisfies these properties:

   (4) $X^{-1}(\bigcup_{i=1}^{\infty} A_i) = \bigcup_{i=1}^{\infty} X^{-1}(A_i)$

   (7) $X^{-1}(\sim A) = \sim X^{-1}(A)$

   You may use these facts without proof in your answer:

(1) If $A \subset B$, then $X^{-1}(A) \subset X^{-1}(B)$

(2) $X^{-1}(A \cup B) = X^{-1}(A) \cup X^{-1}(B)$

(3) $X^{-1}(A \cap B) = X^{-1}(A) \cap X^{-1}(B)$

(5) $X^{-1}\left(\bigcap_{i=1}^{\infty} A_i\right) = \bigcap_{i=1}^{\infty} X^{-1}\left(A_i\right)$

(6) If $h(\omega) = g[X(\omega)]$, then $h^{-1}(B) = X^{-1}[g^{-1}(B)]$

3. Let X be a random variable mapping the measurable space $(\Omega, \mathcal{F})$ onto the measurable space $(\mathcal{X}, \mathcal{A})$. Use the properties stated in Question 2 to show that the collection of sets

$$\mathcal{F}_0 = \left\{F \in \mathcal{F} : F = X^{-1}(A),\ A \in \mathcal{A}\right\}$$

is a σ-algebra.

4. Show that $I_{X^{-1}(F)}(\omega) = I_F[X(\omega)]$.