THE PENNSYLVANIA STATE UNIVERSITY
Department of Economics

Economics 501 — Gallant
Re: Homework 4 — Fall 2014
Due Sept. 23

In discussions with students after class, I think I learned what is causing trouble with this problem:

1. A pair of correlated, six-sided dice are tossed. The random variable X denotes the first toss and the random variable Λ denotes the second; realizations of these tosses are pairs $(x, \lambda) \in \{1,2,3,4,5,6\} \times \{1,2,3,4,5,6\}$. The random variable $D = X - \Lambda$ is the difference. The preimages of D are shown in the first column below and the probabilities $P(D = d)$ are shown in the third column. Fill in the correct entries for the third and fourth columns.

Preimage	d	$P(D=d)$	$P(D=d \mid \Lambda=1)$	$P(D=d \mid \Lambda=2)$
$C_{-5} = \{(1,6)\}$	-5	0	-	-
$C_{-4} = \{(1,5),(2,6)\}$	-4	0	-	-
$C_{-3} = \{(1,4),(2,5),(3,6)\}$	-3	0	-	-
$C_{-2} = \{(1,3),(2,4),(3,5),(4,6)\}$	-2	0	-	-
$C_{-1} = \{(1,2),(2,3),(3,4),(4,5),(5,6)\}$	-1	4/18	-	-
$C_0 = \{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\}$	0	10/18	-	-
$C_1 = \{(2,1),(3,2),(4,3),(5,4),(6,5)\}$	1	4/18	-	-
$C_2 = \{(3,1),(4,2),(5,3),(6,4))\}$	2	0	-	-
$C_3 = \{(4,1),(5,2),(6,3)\}$	3	0	-	-
$C_4 = \{(5,1),(6,2)\}$	4	0	-	-
$C_5 = \{(6,1)\}$	5	0	-	-

What I mean when I tell you that a 1 was thrown on the second toss is that I have told you that the union of the events C_0, C_1, C_2, C_3, C_4, C_5 has occurred.

Let $\mathbb{D} = \{1, 2, 3, 4, 5, 6\}$. What some of you seem to think is that what I have told you is that the event $\mathbb{D} \times \{1\}$ has occurred. Under that interpretation, the set $\mathbb{D} \times \{1\}$ must be added to the σ-algebra before you can work the problem. If so, the σ-algebra is generated by the following mutually exclusive and exhaustive sets

$$
\begin{aligned}
A_1 &= \{(1,1)\} \\
A_2 &= \{(2,1)\} \\
A_3 &= \{(3,1)\} \\
A_4 &= \{(4,1)\} \\
A_5 &= \{(5,1)\} \\
A_6 &= C_5 = \{(6,1)\} \\
A_7 &= C_{-5} = \{(1,6)\} \\
A_8 &= C_{-4} = \{(1,5),(2,6)\} \\
A_9 &= C_{-3} = \{(1,4),(2,5),(3,6)\} \\
A_{10} &= C_{-2} = \{(1,3),(2,4),(3,5),(4,6)\} \\
A_{11} &= C_{-1} = \{(1,2),(2,3),(3,4),(4,5),(5,6)\} \\
A_{12} &= \{(2,2),(3,3),(4,4),(5,5),(6,6)\} \\
A_{13} &= \{(3,2),(4,3),(5,4),(6,5)\} \\
A_{14} &= \{(4,2),(5,3),(6,4))\} \\
A_{15} &= \{(5,2),(6,3)\} \\
A_{16} &= \{(6,2)\}
\end{aligned}
$$

and that I have told you that the union of A_1, A_2, A_3, A_4, A_5, A_6 has occurred.

Now you have a system of equations to solve before you can work the problem:

$$
\begin{aligned}
P(A_1) + P(A_{12}) &= 4/18 \\
P(A_2) + P(A_{13}) &= 10/18 \\
P(A_3) + P(A_{14}) &= 4/18 \\
P(A_4) + P(A_{15}) &= 0 \\
P(A_5) + P(A_{16}) &= 0
\end{aligned}
$$

This system does not have a unique solution because the probabilities of $P(A_1)$, $P(A_2)$, $P(A_3)$, $P(A_{12})$, $P(A_{13})$, $P(A_{14})$ cannot be determined uniquely.

If you absolutely insist on this second view of the problem, then use these probabilities to work it: $P(A_1) = 1/6$, $P(A_2) = 0$, $P(A_3) = 0$. And, explain why the sets A_1 through A_{16} have become the relevant σ-algebra.