THE PENNSYLVANIA STATE UNIVERSITY
Department of Economics

Economics 501 | Gallant
Homework 2 | Fall 2014
Due Sept. 9

1. Let $\mathcal{B}$ be a σ-algebra of subsets of $\mathbb{R}$. Is the collection of sets of the form

$$\left\{(x, y) \in \mathbb{R}^2 \,:\, x - y = d,\, d \in B,\, B \in \mathcal{B}\right\}$$

a σ-algebra?

2. Show the collection of all subsets of Ω is a σ-algebra.

3. Show that the intersection of two σ-algebras is a σ-algebra.

4. Let $\mathcal{A}$ be some collection of sets. Problem 2 implies that there exists at least one σ-algebra that contains $\mathcal{A}$ (Why?). Let $\mathcal{F}$ be the intersection of all σ-algebras that contain $\mathcal{A}$. Show that $\mathcal{F}$ is a σ-algebra. Show that $\mathcal{F}$ is not empty. Why is $\mathcal{F}$ the smallest σ-algebra that contains $\mathcal{A}$?

5. Show that if $F_1, F_2, \ldots$ are mutually exclusive and exhaustive, then the collection of all countable unions plus the empty set is a σ-algebra.

6. Show that if $\mathcal{F}$ is a σ-algebra, then $B \cap \mathcal{F} = \{B \cap F : F \in \mathcal{F}\}$ is a σ-algebra.

   You may assume that $B \in \mathcal{F}$ and $B \neq \emptyset$.

   The definition $B \cap \mathcal{F} = \{B \cap F : F \in \mathcal{F}\}$ is common but ambiguous because it is not clear how one is supposed to take the complement of a set in $B \cap \mathcal{F}$. Implicitly the sample space Ω is getting replaced by B. Therefore, complements are taken relative to B. A relative complement is defined to be the points in B that are not in F and usually denoted by $B \sim F$. One way of taking the relative complement is to take the complement of F relative to Ω and intersect the result with B .