Economics 501   Gallant
Homework 1   Fall 2014
Due Sept. 2

1. Consider the following experiment: One has a coin for which the chance of landing heads is $1/3$ and the chance of landing tails is $2/3$. One tosses that coin three times and observes the number of times heads appears.

   (a) What is a sample space Ω for this experiments that has four elements?

   (b) Write down the σ-algebra $\mathcal{A}$ that contains all events to which probability can be assigned for this experiment.

   (c) Write down the probabilities assigned to all singleton sets in $\mathcal{A}$.

   (d) Assign probabilities to the remaining sets in $\mathcal{A}$ using the fact that the probability assigned to an event $E \in \mathcal{A}$ must be the sum of the probabilities of the singleton sets that comprise E.

2. Prove DeMorgan's laws for countable unions and intersections.

3. Let F_i where $i = 1, 2, \ldots$ be an infinite sequence of events from the sample space Ω. Let F be the set of points that are in all but a finite number of the events F_i. Prove that $F = \bigcup_{k=1}^{\infty} \bigcap_{i=k}^{\infty} F_i$. Make sure that the proof is done carefully: First, take a ω point from F and show that it is in $\bigcup_{k=1}^{\infty} \bigcap_{i=k}^{\infty} F_i$. Secondly, take a point ω from $\bigcup_{k=1}^{\infty} \bigcap_{i=k}^{\infty} F_i$ and show that it is in F.

4. Find the supremum and infimum of the following sets: $\emptyset$, $(-5, 10)$, $(-\infty, \infty)$, $\bigcap_{i=1}^{\infty}\{2/i\}$, $\bigcup_{i=1}^{\infty}\{2/i\}$, $\bigcap_{i=1}^{\infty}[2/i, 2]$, $\bigcup_{i=1}^{\infty}[2/i, 2]$, $\{x : x = 2/i, \ i = 1, 2, \ldots\}$, $\{x : x = -2/i, \ i = 1, 2, \ldots\}$, $\{x : x = 2i, \ i = 1, 2, \ldots\}$, and $\{x : x = -2i, \ i = 1, 2, \ldots\}$.

5. Compute the probability of a win for each of the one roll bets in craps.