

Permit Trading Homework Hints

Environmental Economics

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Fall 2021

The market has the same number of odd and even firms. Thus, we can simplify things by analyzing the case with one odd and one even firm.

1 Short-cut computation of market-clearing permit price

Assume that you know the output of Firm 1, Q_1 , and Firm 2, Q_2 . This will allow you to compute the total amount of abatement A that will be required to support this level of output. Remember to subtract the total number of permits available from the total output $Q = Q_1 + Q_2$ when computing total abatement. Market clearing requires

$$A_1 + A_2 = A \tag{1}$$

The fact that successful trading will equate marginal abatement costs gives the equation

$$6A_1 = 2A_2 \tag{2}$$

A_1 and A_2 can be determined from Equations 1 and 2.

Each firm will choose to abate until the marginal abatement cost equals the price of the permits. Letting P denote the permit price, that means

$$P = 6A_1 = 2A_2.$$

Now you must check to see if the two firms will desire to produce the quantities Q_1 and Q_2 that you guessed given that the permit price is P . If not, you will have to guess and try again.

One way to check is as follows. For Firm i to increase output by one unit, the marginal cost will be $1 + P$, which is the marginal cost of production plus the marginal cost of a permit. If $1 + P$ is less than the selling price of 40 and $Q_i < 20$ then the firm will want to increase output so you should add one to Q_i and try again. If $1 + P$ is greater than 40 you need to subtract one from Q_i and try again.

2 Text-book computation of market-clearing permit price

Permit variables:

P is permit price.

X is permit quantity.

We will assume that Firm 2 sells to Firm 1 because Firm 2 has the lower marginal cost. If this assumption is wrong, X below will be negative, which means Firm 1 sells permits to Firm 2.

Bounds:

$$-10 \leq X \leq 10$$

$$0 \leq P \leq 39$$

$$0 \leq Q_1 \leq 20$$

$$0 \leq Q_2 \leq 20$$

Profits for Firm 1 are revenues - cost - permit purchases - abatement. Abatement for Firm 1 is $Q_1 - 10 - X$. Profits for Firm 2 are revenues - cost + permit sales - abatement. Abatement for Firm 2 is $Q_2 - 10 + X$. Therefore the profit functions are

$$\Pi_1 = 40Q_1 - Q_1 - PX - 3(Q_1 - 10 - X)^2$$

$$\Pi_2 = 40Q_2 - Q_2 + PX - (Q_2 - 10 + X)^2$$

Permit quantity first order conditions given Q_1 and Q_2 :

$$\text{Firm 1's FOC: } \frac{d}{dX} [39Q_1 - PX - 3(Q_1 - 10 - X)^2] = 0$$

$$\text{Firm 2's FOC: } \frac{d}{dX} [39Q_2 + PX - (Q_2 - 10 + X)^2] = 0$$

or

$$\text{Firm 1's FOC: } -P - 6(Q_1 - 10 - X)(-1) = 0$$

$$\text{Firm 2's FOC: } P - 2(Q_2 - 10 + X)(+1) = 0$$

or

$$\text{Firm 1's FOC: } P = 6Q_1 - 60 - 6X$$

$$\text{Firm 2's FOC: } P = 2Q_2 - 20 + 2X$$

Equilibrium:

$$P(Q_1, Q_2) = (3Q_1 + 3Q_2 - 60)/2 \quad 0 \leq P \leq 39$$

$$X(Q_1, Q_2) = (3Q_1 - Q_2 - 20)/4 \quad -10 \leq X \leq 10$$

Derivatives:

$$\frac{dP}{dQ_1} = \frac{3}{2}$$

$$\frac{dP}{dQ_2} = \frac{3}{2}$$

$$\frac{dX}{dQ_1} = \frac{3}{4}$$

$$\frac{dX}{dQ_2} = -\frac{1}{4}$$

Product quantity first order conditions given $P(Q_1, Q_2)$ and $X(Q_1, Q_2)$:

$$\text{Firm 1's FOC: } \frac{d}{dQ_1} \{39Q_1 - X(Q_1, Q_2)P(Q_1, Q_2)\} - 3[Q_1 - 10 - X(Q_1, Q_2)]^2 = 0$$

$$\text{Firm 2's FOC: } \frac{d}{dQ_2} \{39Q_2 + X(Q_1, Q_2)P(Q_1, Q_2)\} - [Q_2 - 10 + X(Q_1, Q_2)]^2 = 0$$

or

$$\text{Firm 1's FOC: } 39 - \frac{dP}{dQ_1}X - P\frac{dX}{dQ_1} - 6(Q_1 - 10 - X)(1 - \frac{dX}{dQ_1}) = 0$$

$$\text{Firm 2's FOC: } 39 + \frac{dP}{dQ_2}X + P\frac{dX}{dQ_2} - 2(Q_2 - 10 + X)(1 + \frac{dX}{dQ_2}) = 0$$

or

$$\text{Firm 1's FOC: } 39 - \frac{3}{2}X - \frac{3}{4}P - 6(Q_1 - 10 - X)(1 - \frac{3}{4}) = 0$$

$$\text{Firm 2's FOC: } 39 + \frac{3}{2}X - \frac{1}{4}P - 2(Q_2 - 10 + X)(1 - \frac{1}{4}) = 0$$

Equilibrium:

$$Q_1 = 36 - \frac{1}{2}P(Q_1, Q_2)$$

$$Q_2 = 36 - \frac{1}{6}P(Q_1, Q_2)$$

If Q_1 exceeds 20, round down to 20. Similarly for Q_2 .

3 Justification of the short-cut computation of the market-clearing permit price

The abatement that each firm does is

$$A_1 = Q_1 - 10 - X$$

$$A_2 = Q_2 - 10 + X$$

If we add these two equations we get the total abatement required

$$A = A_1 + A_2 = Q_1 + Q_2 - 20.$$

From the permit quantity first order conditions given Q_1 and Q_2 we have

$$P = 6(Q_1 - 10 - X) = 6A_1$$

$$P = 2(Q_2 - 10 + X) = 2A_2$$

from which we get abatement marginals are equated $P = 6A_1 = 2A_2$.