

3

alt solution

Profit functions

$$\Pi_1 = P Q_1 + P_x X - C_1(Q_1) \quad Q_1 + X \leq 18$$

$$\Pi_2 = P Q_2 - P_x X - C_2(Q_2) \quad Q_2 \leq 18 + X$$

Assume constraints bind

$$\Pi_1 = P(18 - X) + P_x X - C_1(18 - X)$$

$$\Pi_2 = P(18 + X) - P_x X - C_2(18 + X)$$

$$F_1: -P + P_x - MC_1(18 - X)(-1) = 0$$

$$F_2: P - P_x - MC_2(18 + X)(+1) = 0$$

 \Rightarrow

$$MC_1(18 - X) - MC_2(18 + X) = 0$$

$$4(18 - X) - 2(18 + X) = 0$$

$$2(18 - X) - (18 + X) = 0$$

$$36 - 2X - 18 - X = 0$$

$$18 - 3X = 0$$

$$X = 6$$

Find permit price

$$60 - P_x - MC_1(18+6) = 0$$

$$P_x = 60 - 2(18+6)$$

$$= 60 - 2(24)$$

$$= 60 - 48$$

$$= 12$$